

Ec 11 Homework 2
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CALTECH



1. Suppose demand and supply have constant elasticity equal to 3. What happens to equilibrium price and quantity when the demand increases by 3% and the supply decreases by 3%?
2. Show that elasticity can be expressed as a constant times the change in the log of quantity divided by the change in log of price. (That is, show $\varepsilon = A \frac{d \ln x(p)}{d \ln p}$). Find the constant A .
3. A car manufacturing company employs 100 workers and has two factories, one that produces sedans and one that makes trucks. With m workers, the sedan factory can make m^2 sedans per day. With n workers, the truck factory can make $5n^3$ trucks per day. Graph the production possibilities frontier.
4. In the previous exercise, assume sedans sell for \$20,000 and trucks sell for \$25,000. What assignment of workers maximizes revenue?
5. Define comparative advantage and give an example relating to current events.

1. Suppose demand and supply have constant elasticity equal to 3. What happens to equilibrium price and quantity when the demand increases by 3% and the supply decreases by 3%?

Using the text, demand is $q_d(p) = ap^{-\varepsilon}$ and supply function by $q_s(p) = bp^\eta$. The equilibrium price is $p^* = \left(\frac{a}{b}\right)^{1/\varepsilon+\eta}$ and quantity is $q^* = a^{1/\varepsilon+\eta} b^{\varepsilon/\varepsilon+\eta}$. The parameter a is increased

by 3%, b decreased by 3%, so price changes by $\frac{\Delta p^*}{p^*} = \left(\frac{1.03}{0.97}\right)^{1/\varepsilon+\eta} - 1 \approx 1.0\%$ and

$$\frac{\Delta q^*}{q^*} = 1.03^{1/\varepsilon+\eta} 0.97^{\varepsilon/\varepsilon+\eta} - 1 \approx -0.05\% .$$

Alternatively, we accepted answers which used the approximation that for a small percentage increase Δ in demand, quantity rises by approximately $\frac{\eta\Delta}{\varepsilon+n}$ percent and price rises by approximately $\frac{\Delta}{\varepsilon+n}$ percent. Similarly, for a small percentage increase Δ

in supply, quantity rises by approximately $\frac{\varepsilon\Delta}{\varepsilon+n}$ percent and price falls by approximately $\frac{\Delta}{\varepsilon+n}$ percent.

Using these formulas, we then have

effect of change in demand on price: $3\%/(3+3) = 0.5\%$

effect of change in demand on quantity: $(3*3\%)/(3+3) = 1.5\%$

effect of change in supply on price: $-(-3\%)/(3+3) = 0.5\%$

effect of change in supply on quantity: $(3*-3\%)/(3+3) = -1.5\%$

Summing these effects gives us

Total change in price = $0.5\% + 0.5\% = 1\%$

Total change in quantity = $1.5\% + (-1.5\%) = 0\%$

(Notice that these numbers are in fact approximations of the results obtained using the exact formulas to solve the problem.)

2. Show that elasticity can be expressed as a constant times the change in the log of quantity divided by the change in log of price. (That is, show $\varepsilon = A \frac{d \ln x(p)}{d \ln p}$). Find the constant A .

First note that

$$\frac{d \ln x(p)}{dx(p)} = \frac{1}{x(p)} \Rightarrow d \ln x(p) = \frac{dx(p)}{x(p)}$$

$$\frac{d \ln p}{dp} = \frac{1}{p} \Rightarrow d \ln p = \frac{dp}{p}$$

So that,

$$\frac{d \ln x(p)}{d \ln p} = \frac{p}{x(p)} \frac{dx(p)}{dp} = \frac{px'(p)}{x(p)} = -\varepsilon$$

So $A = -1$.

3. A car manufacturing company employs 100 workers and has two factories, one that produces sedans and one that makes trucks. With m workers, the sedan factory can make m^2 sedans per day. With n workers, the truck factory can make $5n^3$ trucks per day. Graph the production possibilities frontier.

Let S be the number of sedans and T be the number of trucks. We know that $m = \sqrt{S}$

and $n = \sqrt[3]{\frac{T}{5}}$ so that the production possibilities frontier is given by

$$100 = m + n = \sqrt{S} + \sqrt[3]{\frac{T}{5}}$$

4. In the previous exercise, assume sedans sell for \$20,000 and trucks sell for \$25,000. What assignment of workers maximizes revenue?

Revenues can be expressed as $20,000 S + 25,000 T = 20,000 m^2 + 25,000 \cdot 5 \cdot (100 - m)^3$.

The first order condition is then $40,000 m - 375,000 (100 - m)^2 = 0$, which solves for $m = 103.32$ or $m = 96.79$. The only feasible answer is $m = 96.79$, and then $n = 3.21$

But we must also make certain that we have in fact found a maximum. The second order condition is $40,000 + 750,000 (100 - m) > 0$ for $0 \leq m \leq 100$, which means we have actually found a local minimum.

Thus we want to check the endpoints for a solution. When $m = 100$, revenue = 2×10^8 , and when $m = 0$, revenue = 1.25×10^{11} .

Thus the maximum revenue is achieved when $m = 0$ and $n = 100$ (all workers produce trucks).

5. Define comparative advantage and give an example relating to current events.

Comparative advantage is a lower cost of producing one item, expressed in terms of another item. An example would be US automobiles produced in Mexico, IT and telephone support moving to India, or production of movies watched in Europe in the US.