

Ec 11 Final Examination  
 Professor R. Preston McAfee  
 Winter 2005

CALTECH



Instructions: Open book, open notes, no collaboration.  
 Partial credit will be assigned. Please show your work.  
 You may take this test during any consecutive 4 hour period.  
 Due Wednesday, March 16, by 5:00 PM. Please deposit in Box outside Baxter 100.

1. (15 points) A firm discounts future profits of a project at 17%. The firm considers investing in a project that will produce a revenue of \$1,000,000 after one year, growing 5% each year thereafter. What is the most the firm should pay for the project?
2. (20 points) Consider a monopolist with constant marginal cost facing linear demand. A per unit tax of  $t$  is imposed on the monopolist. By how much does the price rise? Assume that the quantity demanded when price is marginal cost plus  $t$  is positive.
3. (20 points) Consider an Edgeworth box, and two goods  $X$  and  $Y$ . Consumer 1 has utility  $\sqrt{x_1 y_1}$  and consumer 2 has utility  $\sqrt{x_2 y_2}$ , where  $x_i$  is the consumption of good  $X$  by consumer  $i$ , and  $y_i$  is the consumption of  $Y$  by consumer  $i$ . Consumer 1 has an endowment of  $(1\frac{1}{2}, \frac{1}{2})$  for the two goods, while consumer 2 has an endowment of  $(\frac{1}{2}, 1\frac{1}{2})$ .
  - a. Compute the contract curve, and find the formula for the individually rational segment of the contract curve.
  - b. What price or prices support an equilibrium of the system? What are the equilibrium quantities?
4. (20 points) Suppose  $u(x) = x^{0.99}$  for a consumer with a current wealth level of \$500,000. A lottery ticket costs \$1 and pays \$1,000,000 with the probability  $\frac{1}{2,000,000}$ . Compute the certainty equivalent of the lottery ticket. It is not expected for you to compute the numbers provided the formula is correct. (Hint: Find the certainty equivalent associated with the gamble with outcomes of \$499,999 and \$1,499,999, the latter with probability  $\frac{1}{2,000,000}$ , then compare that certainty equivalent to the no-lottery outcome of \$500,000.)
5. (10 points) Find all the equilibria of the following game:

		Woman	
		Baseball	Ballet
Man	Baseball	(2,3)	(1,1)
	Ballet	(0,0)	(3,2)

The following questions can be adequately answered with short answers, no more than two or three lines.

6. (5 points, short answer) Describe a negative externality that students sharing living quarters face, and how they might improve the efficiency of their living arrangements.
7. (5 points, short answer) What formula from the Cournot model is used in antitrust analysis? How is it used?
8. (5 points, short answer) What is the value of monitoring employees in terms of incentives and the allocation of risk?

## Answers

1. A firm discounts future profits of a project at 17%. The firm considers investing in a project that will produce a revenue of \$1,000,000 after one year, growing 5% each year thereafter. What is the most the firm should pay for the project?

The present value in millions is

$$PV = \frac{1}{1.17} + \frac{1.05}{1.17^2} + \frac{1.05^2}{1.17^3} + \dots = \frac{1}{1.17} \left( 1 + \frac{1.05}{1.17} + \frac{1.05^2}{1.17^2} + \dots \right)$$
$$\frac{1}{1.17} \left( \frac{1}{1 - \frac{1.05}{1.17}} \right) = \frac{1}{0.12} = 8.33$$

2. Consider a monopolist with constant marginal cost facing linear demand. A per unit tax of  $t$  is imposed on the monopolist. By how much does the price rise?

Let demand be  $q(p) = a - bp$ , and marginal cost  $= c$ . Profits are

$(p - c)q(p) = (p - c)(a - bp)$ , which is maximized at  $p_m = \frac{1}{2}(c + \frac{a}{b})$ . As the tax acts like a marginal cost, the price rises by  $\frac{1}{2}t$ .

3. Consider an Edgeworth box, and two goods  $X$  and  $Y$ . Consumer 1 has utility  $\sqrt{x_1 y_1}$  and consumer 2 has utility  $\sqrt{x_2 y_2}$ , where  $x_i$  is the consumption of good  $X$  by consumer  $i$ , and  $y_i$  is the consumption of  $Y$  by consumer  $i$ . Consumer 1 has an endowment of  $(1\frac{1}{2}, \frac{1}{2})$  for the two goods, while consumer 2 has an endowment of  $(\frac{1}{2}, 1\frac{1}{2})$ .
  - a. Compute the contract curve, and find the formula for the individually rational segment of the contract curve.
  - b. What price or prices support an equilibrium of the system? What are the equilibrium quantities?

A slight variation of the solution in the book gives the contract curve (Section 5.2.6), for  $\alpha = \beta = \frac{1}{2}$ . The solution is

$$\frac{\alpha y}{(1 - \alpha)x} = \frac{\beta(2 - y)}{(1 - \beta)(2 - x)}$$

which yields  $x = y$ .

At the endowment, both parties obtain utility equal to  $\sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}$ . The relevant segment of the contract curve is the diagonal ( $x_1 = y_1$ ) and with  $\frac{1}{2}\sqrt{3} \leq x_1 \leq 2 - \frac{1}{2}\sqrt{3}$ .

An equilibrium has the property that the ratio of the marginal utilities is the budget line for each consumer. This ratio is always one on the contract curve and thus the only possible equilibrium price (for  $y$  in terms of  $x$ ) is 1. With a relative price of 1 and endowments of  $(1\frac{1}{2}, \frac{1}{2})$  and  $(\frac{1}{2}, 1\frac{1}{2})$ , the two parties trade to  $(1, 1)$  and  $(1, 1)$ .

4. Suppose  $u(x) = x^{0.99}$  for a consumer with a current wealth level of \$500,000. A lottery ticket costs \$1 and pays \$1,000,000 with the probability  $\frac{1}{2,000,000}$ .

Compute the certainty equivalent of the lottery ticket. (Hint: Find the certainty equivalent associated with the gamble with outcomes of \$499,999 and \$1,499,999, the latter with probability  $\frac{1}{2,000,000}$ , then compare that certainty equivalent to the no-lottery outcome of \$500,000.)

The expected utility for the consumer of owning the lottery ticket is

$$EU = \frac{1999999}{2000000} (499999^{0.99}) + \frac{1}{2000000} (1499999^{0.99}) = CE^{0.99}$$

The CE of holding the lottery ticket is

$$CE = \left( \frac{1999999}{2000000} (499999^{0.99}) + \frac{1}{2000000} (1499999^{0.99}) \right)^{\frac{1}{0.99}} = 499,999.497$$

Not buying the lottery ticket means a wealth of \$500,000, and thus the lottery ticket has a CE of  $-\$0.503$ . That is, it represents just slight more than a fifty cent loss, which is the expected payout. It is not necessary to actually calculate the numbers provided the formula is written correctly.

5. (5 points) Find all the equilibria of the following game:

		Woman	
		Baseball	Ballet
Man	Baseball	(2,3)	(1,1)
	Ballet	(0,0)	(3,2)

There are two pure strategies: (Baseball, Baseball) and (Ballet, Ballet). In addition, there is a mixed strategy with both randomizing equally between the two outcomes.

The following questions all call for short answers, no more than two or three lines.

6. (10 points, short answer) Describe a negative externality that students sharing living quarters face, and how they might improve the efficiency of their living arrangements.

There are several we discussed in class – noise, cleaning up the common area, etc. Solutions include government, assignment property rights, imposing rules, subsidies and taxes.

7. What formula from the Cournot model is used in antitrust analysis? How is it used?

The HHI is used, which arises from the formula 
$$\sum_{i=1}^n \frac{p(Q) - c'_i(q_i)}{p(Q)} s_i = \frac{HHI}{\varepsilon}$$

that is implied by the first order conditions. The formula is used in assessing the effects of increased industry concentration that would arise from merger.

8. What is the value of monitoring employees in terms of incentives and risk?

Monitoring reduces risk, permitting employee compensation to be more closely based on their efforts.