A Theory of Bilateral Oligopoly*

Kenneth Hendricks
Department of Economics
University of Texas at Austin
Austin TX 78712

R. Preston McAfee
100 Baxter Hall
Humanities & Social Sciences
California Institute of Technology
Pasadena, CA 91125

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Abstract

In horizontal mergers, concentration is often measured with the Hirschmann-Herfindahl Index (HHI). This index yields the price-cost margins in Cournot competition. In many modern merger cases, both buyers and sellers have market power, and indeed, the buyers and sellers may be the same set of firms. In such cases, the HHI is inapplicable. We develop an alternative theory that has similar data requirements as the HHI, applies to intermediate good industries with arbitrary numbers of firms on both sides, and specializes to the HHI when buyers have no market power. The more inelastic is the downstream demand, the more captive production and consumption (not traded in the intermediate market) affects price-cost margins. The analysis is applied to the merger of the gasoline refining and retail assets of Exxon and Mobil in western United States.

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1 Introduction

The seven largest refiners of gasoline on the west coast of the United States account for over 95% of the production of CARB (California Air Resources Board certified) gasoline sold in the region. The seven largest brands of gasoline also accounts for over 97% of retail sales of gasoline. Thus, the wholesale gasoline market on the west coast is composed of a number of large sellers and large buyers who compete against each other in the downstream retail market. What will be the effect of a merger of vertically integrated firms on the wholesale and retail markets? This question has relevance with the mergers of Chevron and Texaco, Conoco and Phillips, Exxon and Mobil, and BP/Amoco and Arco, all of which have been completed in the past decade.

When monopsony or oligopsony faces an oligopoly, most analysts consider that the need for protecting buyers from the exercise of market power is mitigated by the market power of the buyers and vice versa. Thus, even when the buyers and sellers are separate firms, an analysis based on dispersed buyers or dispersed sellers is likely to err. How should antitrust authorities account for the power of buyers and sellers in a bilateral oligopoly market in evaluating the competitiveness of the market? Merging a net buyer with a net seller produces a more balanced firm, bringing what was formerly traded in the intermediate good market inside the firm. Will this vertical integration reduce the exercise of market power and produce a more competitive upstream market? Or will the vertically integrated firm restrict supply to other non-integrated buyers, particularly if they are rivals in the downstream market?

There is a voluminous theoretical literature that address these questions. Most of the literature considers situations in which one or two sellers supply one or two buyers who compete in a downstream market and models their interactions as a bargaining game.[1] Sellers negotiate secret contracts with buyers specifying a quantity to be purchased and transfers to be paid by the buyer. The bilateral bargaining in these models is efficient, so there is no distortions in the wholesale market. Gans [12] uses the model of vertical contracting to derive a concentration index that measures the amount of distortion in the vertical chain as a result of both horizontal concentration amoung buyers and sellers, and the degree of vertical integration. However, the vertical contracting models do not describe

[1] See Rey and Tirole [40] for a survey of this literature. Several of the main papers in this literature are Hart and Tirole [21], McAfee and Schwartz [32], O'Brien and Shaffer [38], Segal [43], and de Fontenay and Gans [9].

intermediate good markets like the wholesale gasoline market in western United States. The market consists of more than two sellers and two buyers, and trades occur at a fixed, and observable, price. Other papers study vertical mergers by assigning the market power either to buyers or to sellers, but not both.[2] These models are excellent for assessing some economic questions, including the incentive to raise rival's cost, the effects of contact in several markets, or the consequences of refusals-to-deal. But, they do not address the implications of bilateral market power that we wish to study in this paper.

Traditional antitrust analysis presumes dispersed buyers. Given such an environment, the Cournot model (quantity competition) suggests that the Hirschman-Herfindahl Index (HHI, which is the sum of the squared market shares of the firms) is proportional to the price-cost margin, which is the proportion of the price that is a markup over marginal cost. Specifically, the HHI divided by the elasticity of demand equals the price-cost margin. The HHI is zero for perfect competition and one for monopoly. The HHI has the major advantage of simplicity and low data requirements. In spite of well-publicized flaws, the HHI continues to be the workhorse of concentration analysis and is used by both the US Department of Justice and the Federal Trade Commission. The HHI is inapplicable, however, to markets where the buyers are concentrated, particularly if they compete in a downstream market.

Our objective in this paper is to offer an alternative to the HHI analysis that applies to homogenous good markets with linear pricing where buyers are concentrated and with (i) similar informational requirements, (ii) the Cournot model as a special case, and (iii) an underlying game as plausible as the Cournot model. The model we offer suffers from the same flaws as the Cournot model. It is highly stylized and static. It uses a "black box" pricing mechanism motivated by the Cournot analysis. Moreover, our model will suffer from the same flaws as the Cournot model in its application to antitrust analysis. Elasticities are treated as constants when they are not, and the relevant elasticities are taken as known. However, the analysis can be applied to markets with arbitrary numbers of sellers and buyers, who individually have the power to influence price, and buyers who may compete against each other in a downstream market. The analysis is simple to apply, and permits the calculation of antitrust effects in a practical way.

Our approach is based on the Klemperer and Meyer [23] market game. In their model,

[2] See for example, Hart and Tirole [21], Ordover, Saloner, and Salop [39], Salinger [41], Salop and Scheffman [42], Bernheim and Whinston [3]. An alternative to assigning the market power to one side of the market is Salinger's sequential model.

sellers submit supply functions and behave strategically, buyers are passiveand report their true demand curves, and price is set to clear the market. We allow the buyers to behave strategically in submitting their demand functions, and apply a similar concept of equilibrium as Klemperer and Meyer. As is well known, supply function models have multiple equilibria. Klemperer and Meyer [23] reduce the multiplicity by introducing stochastic demand, and they show that, if the support is unbounded, then the equilibrium is unique and the equilibrium supply schedule is linear. More recently, Holmberg [15][16] has shown that capacity constraints and a price cap can lead to uniqueness. Green and Newberry [20], Green [18][19], and Akgun [1] obtain uniqueness by restricting the supply schedules to be linear. Our approach is similar but we do not require linearity. In our model, sellers can select from a one-parameter family of schedules indexed by production capacity, and buyers can select from a one-parameter family of schedules indexed by consumption or retailing capacity. Thus, sellers can exaggerate their costs by reporting a capacity that is less than it in fact is, and buyers can understate demand. The main advatage of restricting the selection of schedules is that it allows us to study the strategic interaction between sellers and buyers.

In a traditional assessment of concentration according to the U.S. Department of Justice Merger guidelines, the firms' market shares are intended, where possible, to be shares of capacity. This is surprising in light of the fact that the Cournot model does not suggest the use of capacity shares in the HHI, but rather the share of sales in quantity units (not revenue). Like the Cournot model, the present study suggests using the sales data, rather than the capacity data, as the measure of market share. Capacity plays a role in our theory, and indeed a potential test of the theory is to check that actual capacities, where observed, are close to the capacities consistent with the theory.

The merger guidelines assess the effect of the merger by summing the market shares of the merging parties.[3] Such a procedure provides a useful approximation but is inconsistent with the theory (either Cournot or our theory), since the theory suggests that, if the merging parties' shares don't change, then the prices are unlikely to change as well. We advocate a more computationally-intensive approach, which involves estimating the capacities of the merging parties from the pre-merger market share data. Given those capacities, we then estimate the effect of the merger on the industry, taking into account the incentive of the

[3] Farrell and Shapiro [11] and McAfee and Williams [31] independently criticize the Cournot model while using a Cournot model to address the issue.

merged firm to restrict output (or demand, in the case of buyers). Horizontal mergers among sellers in intermediate input market, where buyers are manufacturing firms, are more likely to raise price and be profitable in our model than in the Cournot model because capacity reports of sellers are typically strategic complements, not strategic substitutes. In wholesale markets, the buyers are retailers, and they typically respond to enhanced seller power by reducing their reported demand, thereby mitigating the effects of the merger and complicating its impact on prices and profits. The model treats horizontal mergers by buyers symmetrically. Vertical mergers in our model generate large efficiency gains because they eliminate two "wedges", the markup by the seller and the markdown by the buyer. Foreclosure effects are important when the merging firms are large.

Structural models of homogenous good markets with dispersed buyers use an ad hoc modification of the Cournot first-order conditions to estimate seller markups. They find that markups are typically much lower than Cournot markups[4] and attribute this finding to the Cournot model's failure to account for a firm's expectations of how rivals will respond to its output choices. In our model, each firm understands that reductions in its supply will be partially offset by increases in the outputs of its rivals. Their response means that the elasticity of each firm's residual demand function exceeds the elasticity of demand, so markups are lower in our model than in the Cournot model. Furthemore, the rivals' responses are determined by their marginal costs, so markups depend upon cost elasticities as well as the demand elasticity. Larger firms have higher markups, and markups are higher in markets where marginal costs are steep. Structural models of vertically related markets typically estimate markups under the assumption that sellers post prices that buyers take as fixed in a sequential vertical-pricing game.[5] In our model, sellers and buyers move simultaneously and the division of rents from market power depends upon cost and demand functions.We investigate the conditions under which the first order conditions of our model can be used to estimate demand and cost parameters.

Our model also provides an interesting alternative for studying spot electricity markets. The operation of these markets closely resemble our market game: generating firms submit supply schedules, buyers report the demands of their retail customers who face regulated prices, and an independent system operator chooses the spot price to equate reported supply

[4]See Bresnahan [7] for a description of the methodology and a survey of a number of empirical studies. More recent studies include Genesove and Mullin [13] and Clay and Troesken [10]

[5]Several recent studies include Goldberg and Verboven [17], Manuszak [29], Mortimer [36], Villas-Boas [46], Villas-Boas and Zhao [45], and Villas-Boas and Hellerstein [47].

to market demand. Empirical studies of these markets have applied supply function models or Cournot models to predict the potential for generating firms to exercise market power in spot electricity markets[6] and to estimate their markups[7]. Our model has the advantages of the supply function model and the simplicity of the Cournot model.

The second section presents a model of intermediate good markets, derives the equilibrium price/cost margins and the value/price margins, which is the equivalent for buyers, for vertically separated markets and for vertically integrated markets. The third section extends the model to spot markets in electricity and wholesale markets in which buyers compete in a downstream market. The fourth section analyzes horizontal and vertical mergers. The fifth section examines identification issues that would arise in trying to apply our model to market data. The sixth section applies the model to the merger of the westcoast assets of Exxon and Mobil to illustrate the plausibility and applicability of the theory. The final section concludes.

2 Intermediate Good Markets

We begin with a standard model of a market for a homogenous intermediate good Q. There are n firms, indexed by i from 1 to n. Each seller i produces output x_i using a constant returns to scale production function with fixed capacity γ_i. Thus, seller i's production costs takes the form

$$C(x_i, \gamma_i) = \gamma_i c\left(\frac{x_i}{\gamma_i}\right), \tag{1}$$

where $c(\cdot)$ is convex and strictly increasing.[8] Each buyer j consumes intermediate output q_j and values that consumption according to a function $V(q_j, k_j)$ where k_j is buyer j's capacity for processing the intermediate output. We assume that V is homogenous of degree one so

[6] Green and Newbery [20], Green [18], Brunekreeft [8] use supply function models to predict markups in the England and Wales Wholesale electricity market; Borenstein and Bushnell [5] apply a Cournot model to predict markups in the California electricity markets, and Bushnell, Mansur, and Saravia [4] also use this model to predict markups in California, New England, and the Pennsylvania, New Jersey, Maryland (PJM) markets.

[7] Empirical studies of markups include Borenstein, Bushnell, and Wolak [6] and Wolak [49] on California, Hortacsu and Puller [22] on Texas, Mansur [28] on PJM, Bushnell, Mansur, and Saravia [4] on California, New England, and PJM, Sweeting [44] and Wolfram [52] on England and New Wales, and Wolak [48, 50, 51] on Australia.

[8] In addition, we assume that $c'(z) \longrightarrow \infty$ as $z \longrightarrow \infty$.

that it can expressed as

$$V(q_j, k_j) = k_j v\left(\frac{q_j}{k_j}\right), \tag{2}$$

where $v(\cdot)$ is concave and strictly increasing.[9] A firm may be both a seller and a buyer, that is, it may produce the intermediate good and also consume it. Such firms are called vertically integrated, although they may be net sellers or net buyers. Letting p denote the market-clearing price in the intermediate good market, the profits to a vertically integrated firm i are given by

$$\pi_i = p\left(x_i - q_i\right) + k_i v\left(\frac{q_i}{k_i}\right) - \gamma_i c\left(\frac{x_i}{\gamma_i}\right). \tag{3}$$

The profits to a firm if it is either a pure seller or a pure buyer can be obtained by setting $q_i = 0$ or $x_i = 0$ respectively.

Markets in which both sellers and buyers exercise market power are called *bilateral oligopoly.* A market with no vertically integrated firms is called a *vertically separated* market. These markets can be further decomposed into oligopoly markets, in which sellers have market power and buyers do not, and oligopsony markets, in which buyers have market power but sellers do not. A market with one or more vertically integrated firms is a *vertically integrated* market.

In what follows, we will need to distinguish between two kinds of intermediate good markets based on the type of buyer. Markets in which buyers are manufacturing firms are called *intermediate input* markets. In these markets, a buyer j combines the intermediate input q_j with capacity k_j using a constant returns to scale technology $F(q_j, k_j)$ to produce a good y_j that it sells at price r.[10] Thus, its revenue function can be expressed as

$$V(q_j, k_j) = r k_i f\left(\frac{q_i}{k_i}\right)$$

A manufacturing firm that has twice the capacity of another firm can produce twice as much at the same average productivity.

Markets in which buyers are retail firms are called *wholesale* markets. In these markets, a buyer j purchases q_j to resell to final consumers at price r. Here $y_j = q_j$ and firm j's

[9] In addition, we assume $v'(z) \longrightarrow 0$ as $z \longrightarrow \infty$.

[10] The production function could include other inputs provided their quantities are proportional to q_i.

valuation of q_j is given by

$$V(q_j, k_j) = rq_j - k_j w\left(\frac{q_j}{k_j}\right) = k_j\left[r\left(\frac{q_j}{k_j}\right) - w\left(\frac{q_j}{k_j}\right)\right]$$

where w represents unit selling costs. A retailer with twice as much selling capacity (e.g., number of stores) can sell twice as much at the same unit cost.

In the Cournot model of oligopoly markets, sellers submit quantities and the market chooses price to equate reported supply to demand. The equilibrium price is equal to each buyer's marginal willingness-to-pay but exceeds each seller's marginal cost of supply. In the standard oligopsony model, buyers submit quantities and the market chooses price to equate reported supply and demand. The equilibrium price is equal to each seller's marginal cost but exceeds each buyer's true willingness to pay. In each of these models, one side of the market is passive and the other side behaves strategically, anticipating the market-clearing mechanism in order to manipulate prices. Our interest, however, is in a market where both buyers and sellers recognize their ability to unilaterally influence the price and behave strategically. In order to model this type of market, we extend the Klemperer and Meyer model, in which sellers behave strategically in submitting supply schedules, to allow buyers to behave strategically in submitting their demand schedules.

In adopting this model, however, we impose some restrictions on the schedules that the firms can report. Sellers have to submit cost schedules which come in the form $\gamma c(x/\gamma)$, and buyers have to submit valuation functions which come in the form $kv(q/k)$. In a mechanism design framework, agents can lie about their type, but they cannot invent an impossible type. The admissible types in our model satisfy (1) and (2), and agents are assumed to be bound by this message space. For sellers, the message space is a one-parameter family of schedules indexed by production capacity, and for buyers, the message space is a one-parameter family of schedules indexed by consumption capacity. Therefore, a seller's type is a capacity γ, a buyer's type is a capacity k,and if the firm is vertically integrated, its type is a pair of capacities (γ, k). Agents (simultaneously) report their types to the market mechanism but, in doing so, they do not have to tell the truth. A seller can exaggerate its costs by reporting a capacity $\widehat{\gamma}$ that is less than it in fact is, and a buyer can understate its willingness to pay by reporting a capacity $\widehat{k}$ that is less than it in fact is.[11] Values and

[11] A buyer's marginal willingness to pay for another unit of input at q units of input is measured by $v'\left(\frac{q}{k}\right)$. Since v is concave, this derivative is increasing in k. Therefore, a buyer understates its willingness to pay

costs are not well-specified at zero capacity. However, the solution can be calculated for arbitrarily small but positive capacities and zero capacity handled as a limit. Firms with zero capacity would then report zero capacity.

Given the agents' reports, the market mechanism chooses price p to equate reported supply and reported demand, and allocates the output efficiently. The solution is characterized by the balance equation,

$$Q = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} x_i = X \tag{4}$$

and the marginal conditions,

$$v'\left(\frac{q_j}{\widehat{k}_j}\right) = p = c'\left(\frac{x_i}{\widehat{\gamma}_i}\right), i, j = 1, .., n. \tag{5}$$

Note that, if everyone tells the truth, then the equilibrium outcome is efficient. Our model can be viewed as turning the market into a black box, as in fact happens in the Cournot model, where the price formation process is not modeled explicitly. Given this black box approach, it seems appropriate to permit the market to be efficient when agents don't, in fact, exercise unilateral power. Such considerations dictate the competitive solution, given the reported types. Any other assumption would impose inefficiencies in the market mechanism, rather than having inefficiencies arise as the consequence of the rational exercise of market power by firms with significant market presence.

Each firm anticipates the market mechanism's decision rule in submitting its reports. From equation (4), it follows that

$$q_i = \frac{\widehat{k}_i Q}{K}, \; x_i = \frac{\widehat{\gamma}_i Q}{\Gamma}, \tag{6}$$

where

$$K = \sum_{i=1}^{n} \widehat{k}_i, \; \Gamma = \sum_{i=1}^{n} \widehat{\gamma}_i. \tag{7}$$

Thus, given the firms' reports, market output $Q(\Gamma, K)$ solves the equation

$$v'\left(\frac{Q}{K}\right) = c'\left(\frac{Q}{\Gamma}\right), \tag{8}$$

by underreporting its capacity.

which depends only on the aggregate production and consumption capacity reports. Market price $p(\Gamma, K)$ is obtained by substituting $Q(\Gamma, K)$ into the marginal conditions of equation (4), and the output is allocated to sellers and buyers using the market share equations of (5).

The firms' actual types are common knowledge to the firms. Thus, in choosing their reports, firms know the true types of other firms. Then the payoff to a vertically integrated firm i from submitting reports $(\widehat{\gamma}_i, \widehat{k}_i)$ is

$$\pi_i = k_i v\left(\frac{\widehat{k}_i Q(\Gamma, K)}{k_i K}\right) - \gamma_i c\left(\frac{\widehat{\gamma}_i Q(\Gamma, K)}{\gamma_i \Gamma}\right) - p(\Gamma, K)Q(\Gamma, K)\left(\frac{\widehat{k}_i}{K} - \frac{\widehat{\gamma}_i}{\Gamma}\right). \tag{9}$$

If firm i is a seller with no consumption capacity, then $k_i = \widehat{k}_i \equiv 0$; similarly, if firm i is a buyer with no production capacity, then $\gamma_i = \widehat{\gamma}_i \equiv 0$. The Nash equilibrium to the market game consists of a profile of reports with the property that (i) each firm correctly guesses the reports of other firms and (ii) no firm has an incentive to submit a different, feasible report.

2.1 Equilibrium

We first derive and discuss equilibrium markups in vertically separated markets. We consider several special cases including the Cournot model. We then derive and discuss the equilibrium markups in vertically integrated markets.

Before stating the theorems, we require some additional notation. The market demand function Q^d is given by $v'(Q^d/K) = P$, so the market elasticity of demand is

$$\varepsilon = -\left(\frac{p}{Q}\right)\left(\frac{dQ^d}{dp}\right) = \frac{-v'(Q/K)}{(Q/K)v''(Q/K)}. \tag{10}$$

Similarly, the market supply function Q^s is given by $c'(Q^s/\Gamma) = P$, so the market elasticity of supply is

$$\eta = \left(\frac{p}{Q}\right)\left(\frac{dQ^s}{dp}\right) = \frac{c'(Q/\Gamma)}{(Q/\Gamma)c''(Q/\Gamma)}. \tag{11}$$

Let σ_i and s_i denote firm i's market share in production and consumption respectively. Given any profile of reports, the market shares are equal to reported capacity shares, that is,

$$\sigma_i = \frac{\widehat{\gamma}_i}{\Gamma},\ s_i = \frac{\widehat{k}_i}{K}. \tag{12}$$

Finally, define

$$c_i' \equiv c'\left(\frac{\sigma_i Q}{\gamma_i}\right),\ v_i' \equiv v'\left(\frac{s_i Q}{k_i}\right). \tag{13}$$

as firm i's equilibrium marginal cost and marginal valuation.

Theorem 1 *Suppose markets are vertically separated. Then*

$$\frac{p - c_i'}{p} = \frac{\sigma_i}{\varepsilon + \eta(1-\sigma_i)}, \tag{14}$$

and

$$\frac{v_i' - p}{p} = \frac{s_i}{\varepsilon(1-s_i) + \eta}. \tag{15}$$

Corollary 2 *(i)* $\dfrac{\widehat{\gamma}}{\gamma}$ *is less than 1 and decreasing in* γ; *(ii)* $\dfrac{\widehat{k}}{k}$ *is less than 1 and decreasing in* k.

The exercise of market power by sellers and buyers creates a double markup problem. Sellers report less than their true capacity, thereby overstating their marginal cost. Since the market mechanism equates price to reported marginal costs, it exceeds each seller's actual marginal cost. Buyers report less than their true capacity, thereby understating their true willingness to pay. As a result, price is less than each buyer's actual marginal willingness to pay. The corollary establishes that, on both sides of the market, the distortion is larger for firms with larger capacities.

As in the standard Cournot model, seller markups are constrained by the elasticity of demand. If demand is elastic, then ε is large and sellers' profit margins are small. However, the seller's margins also depend upon the elasticity of supply. In the Cournot model, if a seller restricts output, market supply falls by the same amount, and the price response depends only upon the elasticity of demand. In a supply function model like ours, if a seller tries to restrict output by reporting a higher marginal cost schedule, the reported market supply shifts to the left causing price to rise, but other sellers move up their reported supply curves, expanding their output. Thus, the fall in market supply is less than the reduction in the seller's output. The price response depends upon the slope of the reported supply curves. If marginal costs are roughly constant (i.e., $c'' \simeq 0$), then η is very large, individual sellers cannot raise price significantly by constricting their supply schedule, and the Bertrand outcome arises. On the other hand, if marginal cost curves are steeply sloped (i.e., $c''/c' \longrightarrow \infty$), then η approaches 0, and the Cournot outcome arises. Since our model treats buyers and sellers symmetrically, the same reasoning applies to buyer markups.

We turn next to vertically integrated markets.

Theorem 3 *Suppose markets are vertically integrated. Then, in any interior equilibrium,* $v_i' = c_i'$ *and*

$$\frac{v_i' - p}{p} = \frac{c_i' - p}{p} = \frac{s_i - \sigma_i}{\varepsilon(1 - s_i) + \eta(1 - \sigma_i)}. \tag{16}$$

There are two immediate observations. First, each vertically integrated firm is technically efficient about its production; that is, its marginal cost is equal to its marginal value. Thus, the firm cannot, in the equilibrium allocation, gain from secretly producing more and consuming that output. This is not to say that the firm could not gain from the ability to secretly produce and consume, for the firm might gain from this ability by altering its reports appropriately. For example, if the firm is a net seller, it will try to raises price by restricting supply and overstating demand. It accomplishes the first by reporting a production capacity $\widehat{\gamma}$ that is less than its actual capacity γ, and the second by reporting a consumption capacity $\widehat{k}$ that exceeds its actual capacity of k. A net buyer does the opposite, reporting higher production capacity and lower consumption capacity than its actual capacities. Second, net buyers value the good more than the price, and net sellers value the good less than price. Thus, net buyers restrict their demand below that which would arise in perfect competition, and net sellers restrict their supply. In both cases, the gain arises because of price effects.

Theorem 4 *The (quantity weighted) average difference between marginal valuations and marginal costs satisfies:*

$$\frac{1}{p}\left(\sum_{i=1}^{n} s_i v_i' - \sum_{i=1}^{n} \sigma_i c_i'\right) = \sum_{i=1}^{n}\left(\frac{(s_i - \sigma_i)^2}{\varepsilon(1 - s_i) + \eta(1 - \sigma_i)}\right). \tag{17}$$

In evaluating proposed horizontal mergers in vertically separated markets, anti-trust agencies (and courts) focus primarily on demand elasticity, the concentration levels in the industry prior to the merger and the predicted change in concentration levels due to the merger, where concentration is measured using the Herfindahl-Hirschman index. This analysis is motivated by the Cournot model. Theorem 4 gives the equivalent of the Hirschman-Herfindahl Index (H) for the present model. We will refer to it as the modified Hirschman-herfindahl Index (MHI). It has the same useful features – it depends only on market shares and elasticities – but there are two important differences. First, it suggests that analysts also need to consider the elasticity of supply in evaluating the competitiveness of the market. Second, it generalizes the analysis to vertically integrated markets and suggests that analysts use the firms' net positions to measure the effects of market power. As noted above, zero net demand causes no inefficiency. Thus, an intermediate good market in which each firm is vertically integrated and supplies only itself is perfectly efficient. However, with even a small but nonzero net demand or supply, size exacerbates the inefficiency.

In this framework, the shares are of production or consumption, and not capacity. The U.S. Department of Justice Merger Guidelines [53] generally calls for evaluation shares of capacity. While our analysis begins with capacities, the shares are actual shares of production (σ_i) or consumption (s_i), rather than the capacity for production and consumption, respectively. Firms may have the same capacity in production and consumption but nevertheless choose to be a net seller or a net buyer depending upon market conditions. The use of actual consumption and production is an advantageous feature of the theory, since these values tend to be readily observed, while capacities are not. Moreover, capacity is often subject to vociferous debate by economic analysts, while the market shares may be more readily observable. Finally, the shares are shares of the total quantity and not revenue shares. However, like the Cournot model, our model is not designed to handle industries with differentiated products, which is the situation where a debate about revenue versus quantity shares arises.

2.2 Intermediate Input Markets with Constant Elasticities

An important special case of our model is one in which value and cost elasticities are constant and buyers are manufacturing firms. Suppose marginal cost is given by

$$c'(z) = az^{\frac{1}{\eta}},$$

where $\eta > 0$, and marginal willingness to pay is given by

$$v'(z) = bz^{-\frac{1}{\varepsilon}},$$

where $\varepsilon > 1$.[12] Given any vector of capacity reports, the market clearing conditions yields closed form solutions for output

$$Q(\Gamma, K) = \Gamma^{\frac{\varepsilon}{\varepsilon+\eta}} K^{\frac{\eta}{\varepsilon+\eta}}$$

and price

$$p(\Gamma, K) = \Gamma^{\frac{-1}{\varepsilon+\eta}} K^{\frac{1}{\varepsilon+\eta}},$$

which facilitates a quantitative assessment of firm misrepresentations and the cost of those misrepresentations. If elasticities vary, the formulae derived from the constant elasticity case apply approximately, with the error determined by the amount of variation in the elasticities.

Let Q_f represent the first best quantity, that which arises when all firms are sincere in their behavior, and p_f be the associated price. Then

Theorem 5 *With constant elasticities, the size of the firms' misrepresentations is given by*

$$\begin{aligned} \frac{\widehat{k}_i}{k_i} &= \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{-\varepsilon} \\ \frac{\widehat{\gamma}_i}{\gamma_i} &= \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{\eta}. \end{aligned} \tag{18}$$

Moreover,

$$\frac{Q_f}{Q} = \left[\sum_{i=1}^{n} s_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{\varepsilon}\right]^{\frac{\eta}{\varepsilon+\eta}} \left[\sum_{i=1}^{n} \sigma_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{-\eta}\right]^{\frac{\varepsilon}{\varepsilon+\eta}} \tag{19}$$

[12] The associated cost and valuation functions are

$$c(z) = \left(\frac{\eta}{\eta+1}\right) z^{\frac{\eta+1}{\eta}}, \; v(z) = \left(\frac{\varepsilon}{\varepsilon-1}\right) z^{\frac{\varepsilon-1}{\varepsilon}}.$$

and

$$\frac{p_f}{p} = \left[\frac{\sum_{i=1}^{n} \sigma_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)} \right)^{-\eta}}{\sum_{i=1}^{n} s_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)} \right)^{\varepsilon}} \right]^{\frac{1}{\varepsilon + \eta}} \tag{20}$$

Equation (17) confirms the intuition that the misrepresentation is largest for the largest net traders, and small for those not participating significantly in the intermediate good market. Indeed, the size of the misrepresentation is proportional to the discrepancy between price and marginal value or cost, as given by Theorem 3, adjusted for the demand elasticity. This is hardly surprising, since the constant demand and supply elasticities insure that marginal values can be converted to misrepresentations in a log-linear fashion.

Equation (18) provides the formula for lost trades. Here there are two effects. Net buyers under-represent their demand, but over-represent their supply. On balance, net buyers under-represent their net demands, which is why the quantity-weighted average marginal value exceeds the quantity-weighted average marginal cost. Equation (18) provides a straightforward means of calculating the extent to which a market is functioning inefficiently, both before and after a merger, at least in the case where the elasticities are approximately constant.

Equation (19) gives the effect of strategic behavior in the model on price. Note that the price can be larger, or smaller, than the efficient full-information price. Market power on the buyer's side (high values of s_i) tend to decrease the price, with buyers exercising market power. Similarly, as σ_i increases, the price tends to rise.

3 Extensions

In this section we consider two extensions of the model. The first is to markets in which the buyers compete in a downstream market. The second is to spot markets like electricity markets in which firms are net traders.

3.1 Downstream Competition

In many, perhaps even most, applications, the assumption that a buyer in the intermediate good market can safely ignore the behavior of other firms in calculating the value of consumption is unfounded. This is particularly true when the buyers are retail firms. In this section, we extend the model to wholesale markets in which retail firms compete in quantities in the downstream market.

Recall that the value of consumption to a retail firm is given by

$$V(q_i, k_i) = r(Q)q_i - k_i w\left(\frac{q_i}{k_i}\right).$$

where $r(Q)$ is the downstream inverse demand and w represents unit selling costs. Firm profits are:

$$\pi_i(\gamma, k) = r(Q)q_i - k_i w\left(\frac{q_i}{k_i}\right) - \gamma_i c\left(\frac{x_i}{\gamma_i}\right) - p(q_i - x_i). \tag{21}$$

As before, we calculate the efficient solution, which satisfies:

$$p = c'(Q/\Gamma) = c'(x_i/\gamma_i) \tag{22}$$

and

$$r(Q) = p + w'(Q/K) = p + w'(q_i/k_i). \tag{23}$$

Let α be the elasticity of downstream demand, and β be the elasticity of the selling cost w. Let θ be ratio of the intermediate good price p to the final good price r. The observables of the analysis will be the market shares (both production, σ_i, and retail, s_i), the elasticity of final good demand, α, of selling cost, β, of production cost, η and the price ratio $\theta = p/r$. It will turn out that the elasticities enter in a particular way, and thus it is useful to define:

$$A = \alpha^{-1}; B = (1-\theta)\beta^{-1}; C = \theta\eta^{-1}. \tag{24}$$

We replicate the analysis of section 3 in the appendix for this more general model. The structure is to use the efficiency equations to construct the value to firm i of reports of $\widehat{k}_i$ and $\widehat{\gamma}_i$. The first order conditions provide necessary conditions for a Nash equilibrium to the reporting game. These first order conditions are used to compute the price/cost margin, weighted by the firm shares. In particular, we look for a modified herfindahl index given by:

$$MHI = \sum_{i-1}^{n} \frac{1}{r}[(r(Q) - p - w_i')s_i + (p - c_i')\sigma_i],$$

where $w_i = w(q_i/\gamma_i)$.

The main theorem characterizes the modified Herfindahl index for an interior solution.

Theorem 6 *In an interior equilibrium,*

$$MHI = \sum_{i=1}^{n} \left[\frac{BC(s_i - \sigma_i)^2 + ABs_i^2(1-\sigma_i) + AC\sigma_i^2(1-s_i)}{A(1-s_i)(1-\sigma_i) + B(1-\sigma_i) + C(1-s_i)} \right] \tag{25}$$

While complex in general, this formula has several important special cases. If $A = 0$, the downstream market has perfectly elastic demand. As a result, r is a constant, and (24) readily reduces to Theorem 4. Note, however, that Theorem 5 does not apply since, in the wholesale market, the market-clearing conditions fail to yield closed form solutions for output and prices.[13] As we shall see in the next section, this distinction between the intermediate input and wholesale market also matters for merger analysis.

When $B = 0$, there is a constant retailing cost w. This case is analogous to Cournot, in that all firms are equally efficient at selling, although the firms vary in their efficiency at producing. In this case, (24) reduces to

$$MHI|_{B=0} = \sum_{i=1}^{n} \left[\frac{AC\sigma_i^2}{A(1-\sigma_i) + C} \right] = \sum_{i=1}^{n} \left[\frac{\theta\sigma_i^2}{\eta(1-\sigma_i) + \theta\alpha} \right]$$

The Herfindahl index reflects the effect of the wholesale market through the elasticity of supply η. If $\eta = 0$, the Cournot HHI arises. For positive η, the possibility of resale increases the price/cost margin. This increase arises because a firm with a large capacity now has an alternative to selling that capacity on the market. A firm with a large capacity can sell some of its Cournot level of capacity to firms with a smaller capacity. The advantage of such sales to the large firm is the reduction in desire of the smaller firms to produce more,

[13] In wholesale markets, $Q(\Gamma, K)$ is defined implicitly by the equilibrium condition

$$\left(\frac{Q}{\Gamma}\right)^{\frac{1}{\eta}} = r - \left(\frac{Q}{K}\right)^{\frac{1}{\beta}}$$

whereas, in intermediate good markets, market clear implies

$$\left(\frac{Q}{\Gamma}\right)^{\frac{1}{\eta}} = \left(\frac{Q}{K}\right)^{-\frac{1}{\varepsilon}},$$

which yields an explicit solubion for $Q(\Gamma, K)$.

which helps increase the retail price. In essence, the larger firms buy off the smaller firms via sales in the intermediate good market, thereby reducing the incentive of the smaller firms to increase their production.

The formula (24) can be decomposed into Herfindahl-type indices for three separate markets: transactions, production and consumption. Note

$$\begin{aligned} MHI &= \sum_{i=1}^{n} \left[\frac{B(1-\sigma_i) + C(1-s_i)}{A(1-s_i)(1-\sigma_i) + B(1-\sigma_i) + C(1-s_i)} \right] \left[\frac{(s_i - \sigma_i)^2}{C^{-1}(1-\sigma_i) + B^{-1}(1-s_i)} \right] \\ &\quad + \sum_{i=1}^{n} \left[\frac{A(1-\sigma_i)(1-s_i)}{A(1-s_i)(1-\sigma_i) + B(1-\sigma_i) + C(1-s_i)} \right] \left[\frac{Bs_i^2}{(1-s_i)} + \frac{C\sigma_i^2}{(1-\sigma_i)} \right] \end{aligned}$$

The modified herfindahl index, MHI, is an average of three separate indices. The first index corresponds to the transactions in the intermediate good market. In form, this term looks like the expression in Theorem 4, adjusted to express the elasticities in terms of the final output prices. The second expression is an average of the indices associated with production and consumption of the intermediate good. These two indices ignore the fact that firms consume some of their own production.

When the downstream market is very elastic, as we have already noted, A is near zero. In this case, the MHI reduces to that of Theorem 4, because elastic demand in the downstream market eliminates downstream effects, so that the only effects arise in the intermediate good market. In contrast, when the downstream market is relatively inelastic, downstream effects dominate, and the MHI is approximately an average of the herfindahl indices for the upstream and downstream markets, viewed as separate markets.

In some sense, these limiting cases provide a resolution of the question of how to treat captive consumption. When demand is very inelastic, as with gasoline in California, then the issue of captive consumption can be ignored without major loss: it is gross production and consumption that matter. In this case, it is appropriate to view the upstream and downstream markets as separate markets and ignore the fact that the same firms may be involved in both. In particular, a merger of a pure producer and a pure retailer should raise minimal concerns. On the other hand, when demand is very elastic (A near zero), gross consumption and gross production can be safely ignored, and the market treated as if the producers and consumers of the intermediate good were separate firms, with net trades in the intermediate good the only issue that arises.[14] Few real world cases are likely to approximate the description of very elastic market demand.[15] However, the case of $A = 0$ also corresponds to the case where the buyers do not compete in a downstream market, and thus may have alternative applications.

In the appendix, we provide the formulae governing the special case of constant elasticities. It is straightforward to compute the reduction in quantity that arises from a concentrated market, as a proportion of the fully efficient, first-best quantity. Moreover, we provide programs which takes market shares as inputs and computes the capital shares of the firms, the quantity reduction and the effects of a merger.[16]

3.2 Electricity Markets

In day-ahead or real-time balancing markets, generating firms submit supply schedules, buyers report the amounts that they need for their retail customers, and an independent system operator chooses price to equate reported supply to market demand. Thus, our market game closely approximates the way in which these markets operate. An important factor determining the generating firms bidding behavior in these markets is their contract positions. With the exception of firms in the California market, generating firms typically sign forward contracts with buyers in which they agree to deliver a fixed amount of electricity at a pre-determined price. Generating firms who have signed such vertical contracts are essentially vertically integrated, and they may be net sellers or net buyers in the spot market. Green [19] and Wolak [48] discuss the theoretical implications of forward contracts and show that they make the spot market more competitive.

Let q_i denote firm i's forward contract quantity and let r denote the contract price. Generating firms typically take short positions in the forward market, in which case, q_i is positive. Firm i's profit from supplying x_i at price p is given by

$$\pi_i = p(x_i - q_i) - v_i c\left(\frac{x_i}{v_i}\right) + rq_i.$$

[14] However, the denominator still depends on gross production and consumption, rather than net production and consumption. This can matter when mergers dramatically change market shares, and even the merger of a pure producer and pure consumer can have an effect.

[15] When market demand is very elastic, it is likely that there are substitutes that have been ignored. It would usually be preferable to account for such substitutes in the market, rather than ignore them.

[16] This program is available on McAfee's website.

The firm's revenues consists of two components: the amount it earns from its contract position and the payment it receives from either reducing its supply below q_i or from increasing its supply above q_i. When it reduces its supply, it is in a net buyer position, buying the reduction in supply at price p from the spot market and selling this amount to its customers at the contract price of r. When it increases its supply, firm i is in a net seller position, selling the increase at price p to retailers. The profit function can also be interpreted as the profits of a vertically integrated firm that sells electricity to its retail customers at a regulated price of r.

We assume that the firms face a downward sloping inverse demand function $p(X)$[17] The operator knows the marginal cost schedules but does not know the capacity that firms have available or at least cannot force them to make all of their capacity available. Firms are asked to report their available generating capacities. Contract positions are common knowledge among the firms. Based on these reports, the operator equates demand to supply and allocates output across firms by equating reported marginal costs so in equilibrium,

$$p(X) = c'\left(\frac{x_i}{\widehat{\nu}}\right)$$

and for $i = 1, .., n$. Note that the allocation rule does not depend upon the firms' contract positions so firms only report their production capacity. Let $\alpha(p)$ is the elasticity of demand at price p and define $s_i = q_i/X$.

Theorem 7 *In an interior equilibrium*

$$\frac{p - c_i'}{p} = \frac{\sigma_i - s_i}{\eta(1 - \sigma_i) + \alpha}.$$

The theorem states that the firm's reported capacity exceeds its true capacity (i.e., marginal cost exceeds reported marginal cost) when the firm produces less than its contract quantity and the opposite is true when it produces more than its contract quantity. It reports truthfully when it is balanced. (If demand is perfectly inelastic, then α is equal to zero in the above formula.) The intuition is that, in former case, the firm is in a net buy position and wants to lower price, whereas in the latter case, it is in a net sell position and wants to raise price. Markups in our model will vary across firms depending upon their

[17] Market demand is downward-sloping in some electricity markets because it is equal to the fixed retail demand less a competitive import supply.

contract positions, and with demand conditions if the elasticity of costs is not constant. Since marginal cost functions in the electricity markets are approximately L-shaped, our model predicts that markups are essentially zero in low demand periods and higher during high demand periods, particularly for large net sellers. This is consistent with the evidence presented in Bushnell, Mansur, and Saravia (BMS) [4].[18] They find that prices are very close to marginal costs during off-peak hours and higher during peak hours.

Our markup equation is closely related to the markup equations that have been estimated in the literature. Wolak [48, 49, 50] assumes that each firm i faces a stochastic residual demand, $RD_i(p, \epsilon)$, and submits a bid schedule that is ex post optimal. That is, for each realization of the random variable ϵ, $x_i(p)$ is a best reply to the ex post residual demand and satisfies the optimality condition

$$\frac{p - c_i'}{p} = \frac{\sigma_i - s_i}{\alpha_i(p, \epsilon)}.$$

where $\alpha_i(p, \varepsilon)$ is the elasticity of the residual demand facing firm i. It incorporates both the elasticity of demand and the aggregate elasticity of supply bid by firm i's rivals and can be estimated from data on the bid schedules of firm i's rivals. In his study of Australian electricity markets, Wolak has data on a firm's contract positions, and he develops a procedure for recovering the firm's cost function from the ex post optimality condition. Hortacsu and Puller [22] show that ex post optimal bid functions are a Bayesian equilibrium when the firms' contract positions are private information and bid strategies are additively separable in the private information. The additivity assumption also implies that the elasticity of the residual demand function does not depend upon ϵ. They exploit the availability of data on the firms' marginal cost and bid schedules in the Texas electricity market to infer the firms' contract positions and then use the markup equations to test the ex post optimality conditions. Sweeting [44] also uses the ex post optimality condition in his study of the Wales electricity market to test the hypothesis of optimal bidding behavior under various assumptions about the firms' contract positions.

[18] Bushnell, Mansur, and Saravia [4] study markups in California, New England, and the Pennsylvania, New Jersey, Maryland (PJM) electricity markets using the Cournot model to predict the potential for generating firms to exercise market power. Borenstein and Bushnell [5] and Borenstein, Bushnell, and Wolak [6] also use the Cournot model to study markups in the California electricty market..

4 Mergers

In this section we examine the equilibrium effects of horizontal and vertical mergers. The constant returns to scale assumption facilitates the study of mergers. It implies that the merger of two firms i and j produces a firm with consumption capacity $k_i + k_j$ and production capacity $\gamma_i + \gamma_j$, and thereby is subject to the same analysis. In what follows, we focus primarily on mergers in vertically separated markets for two reasons. First, previous merger studies typically make this assumption and we wish to compare the results of our analysis to their results. Second, the vertically separated market provides a polar case in which qualitative results can be obtained under the assumption of constant elasticities. The analysis illustrates the economic forces at work in vertically integrated markets, where the impact of mergers depends upon the values of the elasticity parameters and hence requires a more quantitative analysis. We will assume throughout this section that elasticities, including that of downstream demand, are constant.

4.1 Horizontal Mergers

The DOJ's Merger Guidelines estimate the impact of a horizontal merger in oligopoly markets under the assumption that the merged firms do not change their capacity reports.[19] However, as Farrell and Shapiro [11] have observed, this rule ignores the fact that post-merger behavior is likely to be different from pre-merger behavior since the merging firms will internalize the negative externality that their pre-merger actions imposed on each other's profits. An equilibrium analysis is necessary and Farrell and Shapiro provide such an analysis for Cournot oligopoly markets. They investigate the relationship between HHI and consumer and social welfare, and provide necessary and sufficient conditions under which a merger raises price. They also provide sufficient conditions under which profitable mergers raise welfare. Mergers without cost synergies generally raise price but are often not profitable in the Cournot model, since the merging firms reduce output and rival firms respond by expanding their output. McAfee and Williams [31] provide conditions under

[19] Applying this rule to a merger of firms 1 and 2 using our index of market power yields

$$\Delta MHI = (1-\rho_1)\sigma_1^2 + (1-\rho_2)\sigma_2^2 + 2\sigma_1\sigma_2$$

where

$$\rho_i = \frac{\varepsilon + (1-\sigma_1-\sigma_2)}{\varepsilon + (1-\sigma_i)\eta} < 1.$$

Therefore, ΔMHI exceeds ΔHHI (although $MHI < HHI$).

which mergers are profitable for the special case of quadratic costs and linear demand.

We begin our equilibrium analysis of horizontal merger by examining firms' best replies. Substituting equilibrium output and price into the profit function of seller i and taking logs[20], we obtain

$$\log \pi_i = \left(\frac{\eta+1}{\eta}\right)[\log Q(\Gamma, K) - \log \Gamma] + \log\left[\widehat{\nu}_i - \left(\frac{\eta}{\eta+1}\right)\nu_i^{-\frac{1}{\eta}}\widehat{\nu}_i^{\frac{\eta+1}{\eta}}\right]$$

Differentiating yields firm i's best reply, which solves

$$\frac{\widehat{v}_i}{\Gamma}\left[1 - \frac{\partial Q}{\partial \Gamma}\frac{\Gamma}{Q}\right] = \frac{1 - \left(\frac{\widehat{v}_i}{v}\right)^{\frac{1}{\eta}}}{\left[\frac{\eta+1}{\eta} - \left(\frac{\widehat{v}_i}{v_i}\right)^{\frac{1}{\eta}}\right]}. \tag{26}$$

The right-hand-side of equation (25) is decreasing in $\widehat{v}_i$. The two terms on the left-hand side of the equation capture the impact of capacity reports of other firms. The first term is firm i's market share, which falls with reports by other sellers, and the second involves the production capacity output elasticity. An analogous equation determines the buyer's best reply. The key issue that determines whether reports of other firms are strategic substitutes or complements is their impact on the production capacity output elasticity (or consumption capacity output elasticity in the case of buyers). This impact will depend upon the type of market.

In intermediate input markets in which buyers face a constant downstream price, it is easily verified that the output elasticities are constants.[21]

Lemma 8 *Consider a vertically separated, intermediate input market with constant cost and value elasticities and a fixed downstream price. Then (i) the capacity reports by a seller and a buyer are independent of each other and (ii) the capacity reports of any pair of sellers or buyers are strategic complements.*

Lemma 8 implies that intermediate input markets with no vertically integrated firms are

[20] Note that maximizing $\log \pi_i$ is the same as maximizing π_i.

[21] Recall that

$$Q(\Gamma, K) = \Gamma^{\frac{\varepsilon}{\varepsilon+\eta}} K^{\frac{\eta}{\varepsilon+\eta}}.$$

not only structurally separate, they are also strategically separate. It also implies that the reporting game is a log supermodular game.[22]

Theorem 9 *Consider a vertically separated, intermediate input market with constant cost and value elasticities and a fixed downstream price. A horizontal merger among sellers reduces reported production capacity, increases price and decreases output. A horizontal merger among buyers reduces reported consumption capacity, decreases price and output. Horizontal mergers are always profitable for the merging firms.*

We show in the appendix that the best reply of merging firms to pre-merger reports of other firms is always to report a capacity that is less than the sum of their pre-merger reports. It then follows from Lemma 8 that only firms on the same side of the market will respond, and their responses are mutually reinforcing. This leads to the following predictions about the equilibrium impact of horizontal mergers. A merger among sellers reduces reported supply but does not affect reported demand. Hence price increases and output falls. Similarly, a merger among buyers reduces reported demand but does not affect reported supply, so both price and output fall. The merging firms profit from a merger in two ways. First, it gives them more market power to reduce capacity and raise price, and second, other firms on the same side of the market will do the same. The latter effect reflects the key difference between our model and the Cournot model. In our model, best replies are strategic complements whereas, in the Cournot model, they are strategic substitutes. Akgun [1] obtains similar results in a supply function model of oligopoly markets in which the sellers are restricted to reporting linear supply schedules.

In wholesale markets, the equilibrium output elasticities are functions of the aggregate production and retailing capacity.[23] This introduces two new effects into the analysis of a horizontal merger which complicates the analysis of a merger. First, the market is no longer strategically separate. If two sellers merge, buyers will respond. Tedious calculations reveal that the capacity reports of a buyer and a seller are strategic complements as long

[22] Substituting the expression for $Q(\Gamma, K)$ given in the previous footnote, it is easily verified that each seller's (buyer's) profit function is log supermodular in the capacity reports of other sellers (buyers) and independent of the capacity reports of buyers (sellers).

[23] In this case, $Q(\Gamma, K)$ is defined implicitly by

$$\left(\frac{Q}{\Gamma}\right)^{\frac{1}{\eta}} + \left(\frac{Q}{K}\right)^{\frac{1}{\varepsilon}} - Q^{\frac{1}{\alpha}} = 0.$$

as downstream demand is not too inelastic.[24] Thus, when the merging sellers try to reduce their reported capacity, buyers will reduce their capacity reports, thereby lowering reported demand. In this way, the enhanced market power of sellers is mitigated by buyers exercising buyer power. Prices are lower, mergers are less profitable, and inefficiency costs increase. In fact, the strategic complementarity between the buyer and seller reports can lead to nonexistence of an interior solution. For example, if the merger among sellers creates a monopoly, and there is only one buyer who sells at a fixed price of r, it can be shown that the only intersection point of the best replies is (0,0). Clearly, in this case, a merger would not be profitable. Second, the seller reports may no longer be strategic complements. An increase in the reports of other sellers *reduces* the production capacity output elasticity (assuming downstream demand is not too inelastic). This effect dominates the market share effect when sellers have a lot of market power (i.e., η is small).

4.2 Vertical Mergers

There is a voluminous literature on vertical mergers. This literature is primarily concerned with two issues: efficiency and foreclosure. Vertical mergers can generate substantial efficiency gains by eliminating the double markup problem that arises from sellers exercising market power in the upstream market and buyers exercising market power in downstream markets. This is the reason why antitrust agencies are less likely to contest a vertical merger than a horizontal merger. The primary concern that the agencies have about a vertical merger is the risk that the vertically integrated firm will foreclose the intermediate input market to other buyers with whom it may be competing or, more generally, raise their costs by increasing input prices. Analogous effects arise when the vertically integrated firm prevents other sellers from selling input into the intermediate market or forcing them to accept a lower price. The literature mainly considers situations in which one or two sellers supply one or two buyers, and models their interactions as a bargaining game or by assigning market power either to buyers or to sellers, but not both.

In our model, both buyers and sellers exercise market power and all trades occur at a common, fixed price. Sellers misrepresent their costs and buyers misrepresent their willingness to pay and, in equilibrium, the "wedge" between marginal cost and marginal value is the sum of the seller markup and buyer markdown. A vertical merger eliminates the "wedge"

[24] A sufficient condition for strategic complementarity is $\varepsilon \geq \alpha$. However, if α is sufficiently small, the second order conditions will be violated.

between the merging seller and buyer. Consequently, vertical mergers in our model have strong efficiency effects.

To study the foreclosure effect, we focus on intermediate input markets with a constant downstream price.

Theorem 10 *Consider a vertically separated, intermediate input market with constant cost and value elasticities and a fixed downstream price. (i) If the seller is a monopolist and there are at least two buyers, then a vertical merger increases reported demand but not reported supply so price and output increase. (ii) If the buyer is a monopolist and there are at least two sellers, then a vertical merger increases reported supply but not reported demand so output increases and price falls. (iii) If there are at least two sellers and buyers, then a vertical merger increases reported supply and demand, so output increases.*

When the only seller in the market merges with a buyer, it does not change its production capacity report but it overstates consumption capacity. It then follows from Lemma 7 that reported demand increases substantially as other buyers respond by increasing their consumption capacity. Hence, both output and price increases. Similarly, when the only buyer in the market merges with a seller, reported demand does not change but reported supply increases, lowering price and increasing output. Thus, a vertical merger in monopoly or monopsony markets always leads to foreclosure, with the magnitude of the effect depending upon the elasticities of supply and demand and upon market shares. In markets with multiple buyers and sellers, the vertically integrated firm increases both capacity reports, which causes rivals on both sides of the market to increase their capacity reports. Hence, reported supply and demand increases, output increases, but the impact on price is ambiguous. Intuitively, the foreclosure effect is important when the vertically integrated firm is either a large net seller or a large net buyer.

The assumption that the market is vertically separated is crucial to the merger analysis. A vertical merger in a vertically integrated market can lead to a decrease in reported demand or supply. The reason is that reported production capacity and consumption capacity are strategic substitutes for a vertically integrated firm. A similar issue arises with vertical mergers in wholesale markets. Even if the market is vertically separated, increases in reported consumption capacity reduces the reported capacity of sellers and increases in reported production capacity reduces the reported capacity of buyers. Thus, in these cases, the impact of a vertical merger on price and output needs to be computed on a case by case basis.

5 Identification

Suppose a researcher has data on prices and quantities in a market for $t = 1, .., T$ periods and wants to use our model to estimate cost and demand parameters. Under what conditions is the model identified?

In addressing this question, it is useful to begin with the standard model that has been estimated in numerous empirical studies of market power (e.g., Porter [37], Genesove and Mullin [13].Clay and Troesken [10]). The model assumes that the market is vertically separated and that buyers are price-takers. Market demand is given by

$$p_t = P(Q_t, K_t, Z_t, u_{dt}; \delta)$$

where Z_t are observed demand shifters, u_{dt} represents unobserved (to the researcher) demand shocks that are independent over time, and δ are the unknown demand parameters. The pricing equation for sellers is specified as

$$p_t = c'(Q_t, W_t, u_{st}; \phi) - \lambda Q_t \frac{\partial P(Q_t, K_t, Z_t; \delta)}{\partial Q},$$

where W_t are observed factors that shift marginal cost, u_{st} represents unobserved supply shocks, and ϕ are the unknown cost parameters. Here we have assumed that the only aggregate quantity data are available so c' is the marginal cost of the average firm. The parameter λ is known as the "conduct" parameter and interpreted as the average of the firms' conjectures on how aggregate supply will change with an increase in their output. In the Cournot model, rivals cannot react so λ is equal to 1 but, in empirical work, it is often useful to allow markups to vary from the Cournot markups. As is well known (see Bresnahan [7]), the above model is identified if instruments are available for the endogenous variables in the two equations. Shifts in marginal cost can be used to identify the demand parameters, and shifts in demand and in slope of demand can be used to identify the cost and conduct parameters. In the various empirical studies surveyed by Bresnahan [7], estimates of λ range from 0.05 to 0.65. More recently, Genesove and Mullin report estimates of λ for the sugar industry at the turn-of-the-century ranging from 0.038 to 0.10, with the latter computed directly from the data on prices and marginal costs. Clay and Troesken report similarly low estimates for λ in the whiskey industry at the turn-of-the-century. These estimates suggest that dynamic considerations do matter and lead to lower markups.

In our model,

$$\lambda = \frac{\varepsilon}{\varepsilon + (1-\sigma)\eta}$$

where σ denotes the market share of the average firm. It is not a free parameter but in general depends upon the elasticities of reported demand and supply evaluated at the equilibrium market quantity. Note that λ is bounded between 0 and 1, with the upper bound achieved when $\eta = 0$ (i.e., the Cournot case). Hence, our model provides a potential explanation for why estimated markups are typically lower than Cournot markups.

Our model is identified in vertically separated, intermediate input markets with constant cost and value elasticities. More precisely, suppose

$$c'(Q_t, W_t, u_{st}) = W_t^{\phi} Q_t^{1/\eta} \Gamma_t^{-1/\eta} u_{st}$$

and

$$v'(Q_t, Z_t, u_{dt}) = Z_t^{\delta} Q_t^{-1/\varepsilon} K_t^{1/\varepsilon} u_{dt}.$$

where (u_{st}, u_{dt}) are distributed multivariate lognormal with mean zero and covariance Σ. This is the model that Porter estimates under the assumption that buyers are price-takers which, in terms of our model, means that they are reporting their true capacity. As we have observed previously, the elasticities of reported demand and supply are constant in this model, independent of the capacity reports. Furthermore, the argument given for Lemma 7 also implies that capacity reports of sellers and buyers are independent of the observed and unobserved factors shifting demand and supply. Thus, as long as actual capacities are constant over time, Γ and K are constants, as are the firms' market shares. This in turn implies that λ is a constant and that the derivative of the reported demand schedule does not vary with reported buyer capacity. The variation in markups over time is coming from exogenous variation in the observed and unobserved factors but not from the endogenous variables, Γ and K. As a result, changes in W shifts the reported supply but not the reported demand, thereby identifying the demand parameters; changes in Z shifts the reported demand but not the reported supply, thereby identifying the cost parameters.[25]

Our model is not identified if elasticities (and/or slopes of reported demand and supply) depend upon reported capacities and these differ from actual capacities due to the exercise of market power. This will typically be the case in wholesale markets and in vertically integrated markets. In these markets, markups will be a function of K and Γ, which are likely to depend upon the unobserved shocks affecting demand and supply. As a result, λ and P' will vary over time and the variation will be correlated with the unobserved shocks. One could try to find instruments for K and Γ but data on reported capacities are typically not available.

6 Application: The Exxon-Mobil Merger

Our second application is to a merger of Exxon and Mobil's gasoline refining and retailing assets in western United States. The west coast gasoline market is relatively isolated from the rest of the nation, both because of transportation costs,[26] and because of the requirement of gasoline reformulated for lower emission, a type of gasoline known as CARB.

Available market share data is generally imperfect, because of variations due to shutdowns and measurement error, and the present analysis should be viewed as an illustration of the theory rather than a formal analysis of the Exxon-Mobil merger. Nevertheless, we have tried to use the best available data for the analysis. In Table 1, we provide a list of market shares, along with our estimates of the underlying capital shares and the post-merger market shares, which will be discussed below. The data come from Leffler and Pulliam [26].

From Table 1, it is clear that there is a significant market in the intermediate good of bulk (unbranded) gasoline, prior to branding and the addition of proprietary additives. However, the actual size of the intermediate good market is larger than one might conclude

[25] Solving for equilibrium and taking logs, the structural model is given by

$$\begin{aligned}
\log Q_t &= \frac{\varepsilon\delta\eta}{\varepsilon+\eta}\log Z_t - \frac{\phi\varepsilon\eta}{\varepsilon+\eta}\log W_t + \frac{\varepsilon}{\varepsilon+\eta}\log\Gamma + \frac{\eta}{\varepsilon+\eta}\log K + \frac{\varepsilon\eta}{\varepsilon+\eta}\log\frac{u_{dt}}{u_{st}} \\
\log P_t &= \frac{\varepsilon\delta}{\varepsilon+\eta}\log Z_t + \frac{\phi\eta}{\varepsilon+\eta}\log W_t - \frac{\varepsilon}{\varepsilon+\eta}\log\Gamma + \frac{1}{\varepsilon+\eta}\log K + \frac{\varepsilon}{\varepsilon+\eta}\log\frac{u_{dt}}{u_{st}}
\end{aligned}$$

It is straightforward to show that the structural parameters $(\varepsilon, \eta, \delta, \phi)$ can be recovered from parameter estimates of the reduced form regressions of $\log Q_t$ and $\log P_t$ on a constant, $\log Z_t$ and $\log W_t$.

[26] There is currently no pipeline permitting transfer of Texas or Louisiana refined gasoline to California, and the Panama Canal can not handle large tankers, and in any case is expensive. Nevertheless, when prices are high enough, CARB gasoline has been brought from the Hess refinery in the Caribbean.

from Table 1, because firms engage in swaps. Swaps trade gasoline in one region for gasoline in another. Since swaps are balanced, they will not affect the numbers in Table 1.

It is well known that the demand for gasoline is very inelastic. We consider a base case of an elasticity of demand, α, of 1/3. We estimate θ to be 0.7, an estimate derived from an average of 60.1 cents spot price for refined CARB gasoline, out of an average of 85.5 (net of taxes) at the pump in the year 2000.[27] We believe the selling cost to be fairly elastic, with a best estimate of $\beta = 5$. Similarly, by all accounts refining costs are quite inelastic; we use $\eta = 1/2$ as the base case. We will consider the robustness to parameters below, with $\alpha = 1/5$, $\beta = 3$, and $\eta = 1/3$.

Table 2 presents our summary of the Exxon/Mobil merger. The first three columns provide the assumptions on elasticities that define the four rows of calculations. The fourth column provides the markups that would prevail under a fully symmetric and balanced industry, that is, one comprised of fifteen equal sized firms. This is the best outcome that can arise in the model, given the constraint of fifteen firms, and can be used a benchmark. The fifth column considers a world without refined gasoline exchange, in which all fifteen companies are balanced, and is created by averaging production and consumption shares for each firm. This calculation provides an alternative benchmark for comparison, to assess the inefficiency of the intermediate good exchange. The next four columns use the existing market shares, reported in Table 1, as an input, and then compute the price-cost margin and quantity reduction, pre-merger, post-merger, with a refinery sale, or with a sale of retail outlets, respectively.

Table 2 does not use the naive approach of combining Exxon and Mobil's market shares, an approach employed in the Department of Justice Merger Guidelines. In contrast to the merger guidelines approach, we first estimate the capital held by the firms, then combine this capital in the merger, then compute the equilibrium given the post-merger allocation of capital. The estimates are not dramatically different than those that arise using the naive approach of the merger guidelines. To model the divestiture of refining capacity, we combine only the retailing capital of Exxon and Mobil; similarly, to model the sale of retailing, we combine the refining capacity.

The estimated shares of capital are presented in Table 1. These capital shares reflect

[27] We will use all prices net of taxes. As a consequence, the elasticity of demand builds in the effect of taxes, so that a 10% retail price increase (before taxes) corresponds approximately to an 17% increase in the after tax price. Thus, the elasticity of 1/3 corresponds to an actual elasticity of closer to 0.2.

the incentives of large net sellers in the intermediate market to reduce their sales in order to increase the price, and the incentive of large net buyers to decrease their demand to reduce the price. Equilon, the firm resulting from the Shell-Texaco merger, is almost exactly balanced and thus its capital shares are relatively close to its market shares. In contrast, a net seller in the intermediate market like Chevron refines significantly less than its capital share, but retails close to its retail capital share. Arco, a net buyer of unbranded gasoline, sells less than its share from its retail stores, but refines more to its share of refinery capacity. The estimates also reflect the incentives of all parties to reduce their downstream sales to increase the price, an incentive that is larger the larger is the retailer.

The sixth column of Table 2 provides the pre-merger markup, or MHI, and is a direct calculation from equation (24) using the market shares of Table 1. The seventh, eighth and ninth columns combine Exxon and Mobil's capital assets in various ways. The seventh combines both retail and refining capital. The eighth combines retail capital, but leaves Exxon's Benicia refinery in the hands of an alternative supplier not listed in the table. This corresponds to a sale of the Exxon refinery. The ninth and last column considers the alternative of a sale of Exxon's retail outlets. (It has been announced that Exxon will sell both its refining and retailing operations in California.)

Our analysis suggests that without divestiture the merger will, under the hypotheses of the theory, have a small effect on the retail price. In the base case, the markup increases from 20% to 21%, and the retail price increases 1%.[28] Moreover, a sale of a refinery eliminates most the price increase; the predicted price increase is less than a mil. Unless retailing costs are much less elastic than we believe, a sale of retail outlets accomplishes very little. The predicted changes in prices, as a percent of the pre-tax retail price, are summarized in Table 2. The unimportance of retailing is not supported by Hastings [14].

The predicted quantity, as a percentage of the fully efficient quantity, is presented in Table 2, in parentheses. The first three columns present the prevailing parameters. The next six columns correspond to the conceptual experiments discussed above. The symmetric column considers fifteen equal sized firms. The balanced asymmetric column uses the data of Table 1, but averages the refining and retail market shares to yield a no-trade initial solution. The pre-merger column corresponds to Table 1; post-merger combines Exxon and Mobil. Finally, the last two columns consider a divestiture of a refinery and retail assets,

[28] The percentage increase in the retail price can be computed by noting that $p = q^{-A}$.

respectively. We see the effects of the merger through a small quantity reduction. Again, we see that a refinery sale eliminates nearly all of the quantity reduction.

The analysis used the computed market shares rather than the approach espoused by the U.S. Department of Justice Merger Guidelines. Our approach is completely consistent with the theory, unlike the merger guidelines approach, which sets the post-merger share of the merging firms to the sum of their pre-merger shares. This is inconsistent with the theory because the merger will have an impact on all firms' shares. In Table 1, we provide our estimate of the post-merger shares along side the pre-merger shares. Exxon and Mobil were responsible for 18.6% of the refining, and we estimate that the merger will cause them to contract to 17.4%. The other firms increase their share, though not enough to offset the combined firm's contraction.

There is little to be gained by using the naive merger guidelines market shares, because the analysis is sufficiently complicated to require machine-based computation. However, we replicated the analysis using the naive market shares, and the outcomes are virtually identical. Thus, it appears that the naive approach gives the right answer in this application.

7 Conclusion

This paper presents a method for measuring industry concentration in intermediate goods markets. It is especially relevant when firms have captive consumption, that is, some of the producers of the intermediate good use some or all of their own production for downstream sales.

The major advantages of the theory are its applicability to a wide variety of industry structures, its low informational requirements, and its relatively simple formulae. The major disadvantages are the special structure assumed in the theory and the static nature of the analysis. The special structure mirrors Cournot, and thus is subject to the same criticisms as the Cournot model. For all its defects, the Cournot model remains the standard model for antitrust analysis; the present theory extends Cournot-type analysis to a new realm.

We considered the application of the theory to wholesale electricity markets and to the merger of Exxon and Mobil assets in western United States. Several reasonable predictions emerge. In wholesale electricity markets, firm markups are approximately zero during low demand periods, high during high demand periods, and vary depending upon the firm's net position and market share. In west coast gasoline markets, the industry produces around

95% of the efficient quantity and the merger reduces quantity by a small amount, around 0.3%. The price-cost margin is on the order of 20% and rises by a percentage point or two. A sale of Exxon's refinery eliminates nearly all of the predicted price increase. This last prediction arises because retailing costs are relatively elastic, so that firms are fairly competitive downstream. Thus, the effects of industry concentration arise primarily from refining, rather than retail. Hence the sale of a refinery (Exxon and Mobil have one refinery each) cures most of the problem associated with the merger. The naive approach based purely on market shares gives answers similar to the more sophisticated approach of first computing the capital levels, combining the capital of the merging parties, then computing the new equilibrium market shares. Finally, it is worth noting that the computations associated with the present analysis are straightforward, and run in a few seconds on a modern PC.

As with Cournot analysis, the static nature of the theory is problematic. In some industries, entry of new capacity is sufficiently easy that entry would undercut any exercise of market power. The present theory does not accommodate entry, and thus any analysis would need a separate consideration of the feasibility and likelihood of timely entry.[29] When entry is an important consideration, the present analysis provides an upper bound on the ill-effects merger.

Another limitation of the theory is the restriction to homogenous good markets. An extension of the theoretical approach to markets in which sellers offer differentiated goods would be challenging but worth exploring.

[29] McAfee, Simons and Williams [33] present a Cournot-based merger evaluation theory that explicitly accommodates entry in the analysis.

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Table 1: Approximate Market Shares, West Coast CARB Gasoline Post-Merger Numbers in Parentheses					
Company	i	Refining Market Share (σ_i)	Refining Capital Share	Retail Market Share (s_i)	Retail Capital Share
Chevron	1	26.4 (26.6)	29.5 (29.5)	19.2 (19.5)	19.0 (19.0)
Tosco	2	21.5 (21.7)	21.7 (21.7)	17.8 (18.0)	17.8 (17.8)
Equilon	3	16.6 (16.7)	16.1 (16.1)	16.0 (16.2)	16.0 (16.0)
Arco	4	13.8 (13.9)	13.0 (13.0)	20.4 (20.7)	22.0 (22.0)
Mobil	5	7.0 (13.3)	6.2 (12.4)	9.7 (17.5)	9.3 (17.8)
Exxon	6	7.0 (0.0)	6.2 (0.0)	8.9 (0.0)	8.5 (0.0)
Ultramar	7	5.4 (5.4)	4.7 (4.7)	6.8 (6.9)	6.4 (6.4)
Paramount	8	2.3 (2.3)	2.0 (2.0)	0.0 (0.0)	0.0 (0.0)
Kern	9	0.0 (0.0)	0.0 (0.0)	0.3 (0.3)	0.27 (0.27)
Koch	10	0.0 (0.0)	0.0 (0.0)	0.2 (0.2)	0.18 (0.18)
Vitol	11	0.0 (0.0)	0.0 (0.0)	0.2 (0.2)	0.18 (0.18)
Tesoro	12	0.0 (0.0)	0.0 (0.0)	0.2 (0.2)	0.18 (0.18)
PetroDiamond	13	0.0 (0.0)	0.0 (0.0)	0.1 (0.1)	0.09 (0.09)
Time	14	0.0 (0.0)	0.0 (0.0)	0.1 (0.1)	0.09 (0.09)
Glencoe	15	0.0 (0.0)	0.0 (0.0)	0.1 (0.1)	0.09 (0.09)

Table 2: Analysis of Exxon – Mobil Merger								
Cases			Markup as a Percent of Retail Price (Quantity as a Percent of Fully Efficient Quantity in Parentheses)					
α	β	η	Symmetric	Balanced Asymmetric	Pre-merger Markup	Post-merger Markup	Refinery Sale	Retail Sale
1/3	5	½	6.9 (98.4)	18.4 (95.3)	20.0 (94.6)	21.3 (94.3)	20.1 (94.6)	21.2 (94.3)
1/5	5	½	7.9 (98.7)	21.6 (96.0)	23.6 (95.4)	25.2 (95.2)	23.7 (95.4)	25.2 (95.2)
1/3	3	½	7.0 (98.4)	18.7 (95.3)	20.3 (94.6)	21.7 (94.3)	20.5 (94.6)	21.6 (94.4)
1/3	5	1/3	8.7 (98.2)	23.0 (94.6)	25.1 (93.8)	26.7 (93.5)	25.2 (93.8)	26.7 (93.5)

Table 3: Analysis of Exxon – Mobil Merger					
Cases			Expected Percentage Quantity Decrease (Percentage Price Increase in Parentheses)		
α	β	η	Full Merger	Refinery Sale	Retail Sale
1/3	5	½	0.31 (0.94)	0.03 (0.09)	0.30 (0.90)
1/5	5	½	0.27 (1.36)	0.02 (0.11)	0.25 (1.29)
1/3	3	½	0.32 (0.97)	0.05 (0.15)	0.30 (0.89)
1/3	5	1/3	0.35 (1.06)	0.03 (0.08)	0.34 (1.03)

Appendix

Proof of Theorems 1 and 3: Before proceeding, note that differentiating the equilibrium condition in equation (7) implies that

$$\left(\frac{K}{Q}\right)\frac{\partial Q}{\partial K}=\frac{\varepsilon^{-1}}{\varepsilon^{-1}+\eta^{-1}},\ \left(\frac{\Gamma}{Q}\right)\frac{\partial Q}{\partial \Gamma}=\frac{\eta^{-1}}{\varepsilon^{-1}+\eta^{-1}},\ and\ \left(\frac{K}{P}\right)\frac{\partial P}{\partial K}=-\left(\frac{\Gamma}{P}\right)\frac{\partial P}{\partial \Gamma}=\frac{(\eta\varepsilon)^{-1}}{\varepsilon^{-1}+\eta^{-1}}.$$

Differentiating the firm's profit function in equation (8) and substituting the relations given above yields the following first order conditions:

$$\frac{\partial \pi_i}{\partial \hat{k}_i}=\left(v'\left(\frac{s_iQ}{k_i}\right)-p\right)\left(Q\left(\frac{1}{K}-\frac{\hat{k}_i}{K^2}\right)+s_i\frac{\partial Q}{\partial K}\right)+\left(p-c'\left(\frac{\sigma_iQ}{\gamma_i}\right)\right)\left(\sigma_i\frac{\partial Q}{\partial K}\right)-Q(s_i-\sigma_i)\frac{\partial p}{\partial K}$$

$$=\frac{Q}{K}\left[(v_i'-p)\left((1-s_i)+s_i\frac{\varepsilon^{-1}}{\varepsilon^{-1}+\eta^{-1}}\right)+(p-c_i')\left(\sigma_i\frac{\varepsilon^{-1}}{\varepsilon^{-1}+\eta^{-1}}\right)-p(s_i-\sigma_i)\frac{(\eta\varepsilon)^{-1}}{\varepsilon^{-1}+\eta^{-1}}\right].$$

Similarly,

$$\frac{\partial \pi_i}{\partial \hat{\gamma}_i}=\left(v'\left(\frac{s_iQ}{k_i}\right)-p\right)\left(s_i\frac{\partial Q}{\partial \Gamma}\right)+\left(p-c'\left(\frac{\sigma_iQ}{\gamma_i}\right)\right)\left(Q\left(\frac{1}{\Gamma}-\frac{\hat{\gamma}_i}{\Gamma^2}\right)+\sigma_i\frac{\partial Q}{\partial \Gamma}\right)-Q(s_i-\sigma_i)\frac{\partial p}{\partial \Gamma}$$

$$=\frac{Q}{\Gamma}\left[(v_i'-p)\left(s_i\frac{\eta^{-1}}{\varepsilon^{-1}+\eta^{-1}}\right)+(p-c_i')\left(1-\sigma_i+\sigma_i\frac{\eta^{-1}}{\varepsilon^{-1}+\eta^{-1}}\right)+p(s_i-\sigma_i)\frac{(\eta\varepsilon)^{-1}}{\varepsilon^{-1}+\eta^{-1}}\right].$$

Thus,

$$\frac{K}{Q}\frac{\partial \pi_i}{\partial \hat{k}_i}+\frac{\Gamma}{Q}\frac{\partial \pi_i}{\partial \hat{\gamma}_i}=v_i'-c_i'.$$

In an interior equilibrium, then, $v_i'-c_i'=0$. Either of the first order conditions yields (15).

The Case of $s_i=0$, $\sigma_i>0$.

If $\sigma_i>0$, the first order condition on $\hat{\gamma}_i$ holds with equality. Consequently, using $s_i=0$,

$$0=\frac{Q}{\Gamma}\left[(p-c_i')\left(1-\sigma_i+\sigma_i\frac{\eta^{-1}}{\varepsilon^{-1}+\eta^{-1}}\right)+p(0-\sigma_i)\frac{(\eta\varepsilon)^{-1}}{\varepsilon^{-1}+\eta^{-1}}\right]$$

This yields

$$\frac{p - c_i'}{p} = \frac{\sigma_i}{\varepsilon + \eta(1-\sigma_i)},$$

a formula that respects (15) (for s_i=0). In addition, we have

$$0 \geq \left.\frac{\partial \pi_i}{\partial \hat{k}_i}\right|_{s_i=0},$$

which gives

$$\frac{v'(0) - p}{p} \leq \frac{-\sigma_i}{\varepsilon + \eta(1-\sigma_i)}.$$

Summarizing,

$$\frac{v_i' - p}{p} \leq \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}, \textit{ with equality if } s_i > 0.$$

and

$$\frac{p - c_i'}{p} \leq \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}, \textit{ with equality if } \sigma_i > 0.$$

Suppose k_i=0. Then $kv(q/k) = \dfrac{v(q/k)}{1/k} \approx qv'(q/k) \xrightarrow[k\to 0]{} 0$. Thus, an agent with k_i=0 will report $\hat{k}_i = 0$. Similarly, an agent with γ_i=0 produces zero. This yields (13) and (14). Q.E.D.

Proof of Corollary 2:

Suppose $\widehat{\gamma}_i > \widehat{\gamma}_j$. Then equation (5) implies that $x_i > x_j$ and hence that $\sigma_i > \sigma_j$. It then follows from the first-order conditions of firms i and j that

$$c'\left(\frac{x_i}{\gamma_i}\right) < c'\left(\frac{x_j}{\gamma_j}\right) \Longrightarrow \frac{x_i}{\gamma_i} < \frac{x_j}{\gamma_j} \Longrightarrow \frac{\widehat{\gamma}_i}{\gamma_i} < \frac{\widehat{\gamma}_j}{\gamma_j} \textit{ and } \gamma_i > \gamma_j.$$

The reasoning for buyers is similar. Q.E.D.

Proof of Theorem 4: It is readily checked that the following substitutions hold, even when a share is zero.

$$\sum_{i=1}^{n} v_i' s_i - \sum_{i=1}^{n} c_i' \sigma_i = \sum_{i=1}^{n} (v_i' - p) s_i + \sum_{i=1}^{n} (p - c_i') \sigma_i$$

$$= \sum_{i=1}^{n} (v_i' - p) s_i + \sum_{i=1}^{n} (p - v_i') \sigma_i = \sum_{i=1}^{n} (v_i' - p) s_i - \sum_{i=1}^{n} (v_i' - p) \sigma_i$$

$$= \sum_{i=1}^{n} (v_i' - p)(s_i - \sigma_i) = \sum_{i=1}^{n} \frac{p(s_i - \sigma_i)^2}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}. \qquad \text{Q.E.D.}$$

Proof of Theorem 5: Applying (7), (15), (17):

$$\left(\frac{s_i Q}{k_i}\right)^{-\frac{1}{\varepsilon}} = v'(\frac{s_i Q}{k_i}) = p\left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right) = v'\left(\frac{Q}{\sum_{i=1}^{n} \hat{k}_i}\right)\left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right).$$

This readily gives the first part of (18); the second half is symmetric.

Rewrite (18) to obtain

$$k_i = \hat{k}_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{\varepsilon}.$$

Thus

$$\frac{k_i}{\sum_{j=1}^{n} \hat{k}_j} = s_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{\varepsilon}.$$

Thus

$$\frac{\sum_{i=1}^{n} k_i}{\sum_{i=1}^{n} \hat{k}_i} = \sum_{i=1}^{n} s_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{\varepsilon}.$$

A similar calculation gives

$$\frac{\sum_{i=1}^{n} \gamma_i}{\sum_{i=1}^{n} \hat{\gamma}_i} = \sum_{i=1}^{n} \sigma_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{-\eta}.$$

Note that, with constant elasticity, actual quantity is

$$Q = \left(\sum_{i=1}^{n} \hat{k}_i\right)^{\frac{\eta}{\varepsilon+\eta}} \left(\sum_{i=1}^{n} \hat{\gamma}_i\right)^{\frac{\varepsilon}{\varepsilon+\eta}},$$

and

$$Q_f = \left(\sum_{i=1}^{n} k_i\right)^{\frac{\eta}{\varepsilon+\eta}} \left(\sum_{i=1}^{n} \gamma_i\right)^{\frac{\varepsilon}{\varepsilon+\eta}}.$$

Substitution gives (18). One obtains (19) from

$$\frac{p}{p_f} = \frac{v'(Q/\sum_{i=1}^{n} \hat{k}_i)}{v'(Q_f/\sum_{i=1}^{n} k_i)} = \left(\frac{Q}{Q_f} \frac{\sum_{i=1}^{n} k_i}{\sum_{i=1}^{n} \hat{k}_i}\right)^{\frac{-1}{\varepsilon}} = \left(\frac{Q_f}{Q} \frac{\sum_{i=1}^{n} \hat{k}_i}{\sum_{i=1}^{n} k_i}\right)^{\frac{1}{\varepsilon}}$$

$$= \left[\left(\frac{\sum_{i=1}^{n} k_i}{\sum_{i=1}^{n} \hat{k}_i}\right)^{\frac{\eta}{\varepsilon+\eta}} \left(\frac{\sum_{i=1}^{n} \gamma_i}{\sum_{i=1}^{n} \hat{\gamma}_i}\right)^{\frac{\varepsilon}{\varepsilon+\eta}} \frac{\sum_{i=1}^{n} \hat{k}_i}{\sum_{i=1}^{n} k_i}\right]^{\frac{1}{\varepsilon}} = \left[\left(\frac{\sum_{i=1}^{n} \gamma_i}{\sum_{i=1}^{n} \hat{\gamma}_i}\right)\left(\frac{\sum_{i=1}^{n} \hat{k}_i}{\sum_{i=1}^{n} k_i}\right)\right]^{\frac{1}{\varepsilon+\eta}}$$

$$= \left[\frac{\sum_{i=1}^{n} \sigma_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{-\eta}}{\sum_{i=1}^{n} s_i \left(1 + \frac{s_i - \sigma_i}{\varepsilon(1-s_i) + \eta(1-\sigma_i)}\right)^{\varepsilon}}\right]^{\frac{1}{\varepsilon+\eta}}.$$

Q.E.D.

Proof of Theorem 6:

Using the market calculations (21) and (22), rewrite profits to obtain

$$\pi_i = r(Q)q_i - k_i w\left(q_i/k_i\right) - \gamma_i c\left(x_i/\gamma_i\right) - p(q_i - x_i)$$

$$= (r(Q) - p)q_i - k_i w\left(q_i/k_i\right) + px_i - \gamma_i c\left(x_i/\gamma_i\right)$$

$$= w'\left(Q/K\right)q_i - k_i w\left(q_i/k_i\right) + c'\left(Q/\Gamma\right)x_i - c\left(x_i/\gamma_i\right)$$

$$= w'\left(\frac{Q}{K}\right)\frac{\hat{k}_i}{K}Q - k_i w\left(\frac{\hat{k}_i}{K}\frac{Q}{k_i}\right) + c'\left(\frac{Q}{\Gamma}\right)\frac{\hat{\gamma}_i}{\Gamma}Q - \gamma_i c\left(\frac{\hat{\gamma}_i}{\Gamma}\frac{Q}{\gamma_i}\right).$$

The equilibrium quantity is given by

$$r(Q) - w'(Q/K) - c'(Q/\Gamma) = 0.$$

From this equation, and applying (23), it is a routine computation to show:

$$\frac{K}{Q}\frac{dQ}{dK} = \frac{K}{Q}\frac{-(w'')Q/K^2}{r' - w''/K - c''/\Gamma} = \frac{(r-p)\beta^{-1}}{r\alpha^{-1} + (r-p)\beta^{-1} + p\eta^{-1}} = \frac{B}{A+B+C}.$$

Similarly,

$$\frac{\Gamma}{Q}\frac{dQ}{d\Gamma} = \frac{C}{A+B+C}.$$

Differentiating π_i, and using the analogous notation for

$$0 = \frac{K}{Q}\frac{\partial \pi_i}{\partial \hat{k}_i} = (w' - w_i')(1 - s_i + s_i\frac{K}{Q}\frac{dQ}{dK}) - w''\frac{s_i Q}{K} + (c' - c_i')\sigma_i\frac{K}{Q}\frac{dQ}{dK} + (w'' s_i\frac{Q}{K} + c''\sigma_i\frac{Q}{\Gamma})\frac{K}{Q}\frac{dQ}{dK}$$

$$= (w' - w_i')(1 - s_i + s_i\frac{K}{Q}\frac{dQ}{dK}) + (c' - c_i')\sigma_i\frac{K}{Q}\frac{dQ}{dK} - \beta^{-1}w' s_i + (\beta^{-1}w' s_i + \eta^{-1}c'\sigma_i)\frac{K}{Q}\frac{dQ}{dK}$$

$$= (w' - w_i')(1 - s_i + s_i\frac{K}{Q}\frac{dQ}{dK}) + (c' - c_i')\sigma_i\frac{K}{Q}\frac{dQ}{dK} - r\left(Bs_i - (Bs_i + C\sigma_i)\frac{K}{Q}\frac{dQ}{dK}\right)$$

$$= (w' - w_i')(1 - s_i + s_i\frac{K}{Q}\frac{dQ}{dK}) + (c' - c_i')\sigma_i\frac{K}{Q}\frac{dQ}{dK} - r\left(Bs_i\left(1 - \frac{K}{Q}\frac{dQ}{dK}\right) - C\sigma_i\frac{K}{Q}\frac{dQ}{dK}\right)$$

Similarly, and symmetrically,

$$0 = \frac{\Gamma}{Q}\frac{\partial \pi_i}{\partial \hat{\gamma}_i} = (w' - w_i')s_i\frac{\Gamma}{Q}\frac{dQ}{d\Gamma} + (c' - c_i')(1 - \sigma_i + \sigma_i\frac{\Gamma}{Q}\frac{dQ}{d\Gamma}) - r\left(C\sigma_i\left(1 - \frac{\Gamma}{Q}\frac{dQ}{d\Gamma}\right) - Bs_i\frac{\Gamma}{Q}\frac{dQ}{d\Gamma}\right).$$

These equations can be expressed, substituting the elasticities with respect to capacity, as

$$\begin{bmatrix} A+B+C-s_i(A+C) & B\sigma_i \\ Cs_i & A+B+C-\sigma_i(A+B) \end{bmatrix}\begin{pmatrix} (w'-w_i')/r \\ (c'-c_i')/r \end{pmatrix} = \begin{pmatrix} Bs_i(A+C) - BC\sigma_i \\ c\sigma_i(A+B) - BCs_i \end{pmatrix}$$

The determinant of the left hand side matrix is given by

$$DET = [A+B+C-s_i(A+C)][A+B+C-\sigma_i(A+B)]-BCs_i\sigma_i$$

$$= (A+B+C)[(A+B+C)(1-s_i)(1-\sigma_i)+Bs_i(1-\sigma_i)+C(1-s_i)\sigma_i]$$

$$= (A+B+C)[A(1-s_i)(1-\sigma_i)+B(1-\sigma_i)+C(1-s_i)].$$

Thus,

$$\begin{pmatrix} (w'-w_i')/r \\ (c'-c_i')/r \end{pmatrix} = \frac{1}{DET}\begin{bmatrix} A+B+C-\sigma_i(A+B) & -B\sigma_i \\ -Cs_i & A+B+C-s_i(A+C) \end{bmatrix}\begin{pmatrix} Bs_i(A+C)-BC\sigma_i \\ c\sigma_i(A+B)-BCs_i \end{pmatrix}$$

$$= \frac{1}{DET}\begin{pmatrix} (A+B+C-\sigma_i(A+B))(Bs_i(A+C)-BC\sigma_i)-B\sigma_i(c\sigma_i(A+B)-BCs_i) \\ -Cs_i(Bs_i(A+C)-BC\sigma_i)+(A+B+C-s_i(A+C))(c\sigma_i(A+B)-BCs_i) \end{pmatrix}$$

$$= \frac{A+B+C}{DET}\begin{pmatrix} (Bs_i(A+C)-BC\sigma_i)-ABs_i\sigma_i \\ (C\sigma_i(A+B)-BCs_i)-ACs_i\sigma_i \end{pmatrix} = \frac{A+B+C}{DET}\begin{pmatrix} B[C(s_i-\sigma_i)+As_i(1-\sigma_i)] \\ C[B(\sigma_i-s_i)+A\sigma_i(1-s_i)] \end{pmatrix}.$$

Thus,

$$MHI = \sum_{i=1}^{n}\left(\frac{(r(Q)-p-w_i')s_i+(p-c_i')\sigma_i}{r}\right)$$

$$= \sum_{i=1}^{n}\left(\frac{(w'-w_i')s_i+(c'-c_i')\sigma_i}{r}\right)$$

$$= \sum_{i=1}^{n}\left[\left(\frac{A+B+C}{DET}\right)\left(s_iB[C(s_i-\sigma_i)+As_i(1-\sigma_i)]+\sigma_iC[B(\sigma_i-s_i)+A\sigma_i(1-s_i)]\right)\right]$$

$$= \sum_{i=1}^{n}\left[\left(\frac{A+B+C}{DET}\right)\left[BC(s_i-\sigma_i)^2+ABs_i^2(1-\sigma_i)+AC\sigma_i^2(1-s_i)\right]\right]$$

$$= \sum_{i=1}^{n}\frac{BC(s_i-\sigma_i)^2+ABs_i^2(1-\sigma_i)+AC\sigma_i^2(1-s_i)}{A(1-s_i)(1-\sigma_i)+B(1-\sigma_i)+C(1-s_i)}. \qquad Q.E.D.$$

The Constant Elasticity Case:

$$\begin{pmatrix} w' - w_i' \\ c' - c_i' \end{pmatrix} = r\begin{pmatrix} \dfrac{B[C(s_i - \sigma_i) + As_i(1-\sigma_i)]}{A(1-s_i)(1-\sigma_i) + B(1-\sigma_i) + C(1-s_i)} \\ \dfrac{C[B(\sigma_i - s_i) + A\sigma_i(1-s_i)]}{A(1-s_i)(1-\sigma_i) + B(1-\sigma_i) + C(1-s_i)} \end{pmatrix} \equiv r\begin{pmatrix} \psi_i \\ \chi_i \end{pmatrix}.$$

Then

$$\left(\frac{s_i Q}{k_i}\right)^{\frac{1}{\beta}} = w_i' = w' - r\psi_i = r(1-\theta-\psi_i)$$

or,

$$k_i = s_i Q[r(1-\theta-\psi_i)]^{-\beta}$$

Similarly,

$$\gamma_i = \sigma_i Q[r(\theta-\chi_i)]^{-\eta}$$

The efficient solution satisfies $r - w'\left(Q_f \big/ \sum k_i\right) - c'\left(Q_f \big/ \sum \gamma_i\right) = 0$, or

$$0 = Q_f^{-\frac{1}{\alpha}} - \left(\frac{Q_f}{Q}\right)^{\frac{1}{\beta}} r\left(\sum_{i=1}^{n} s_i[1-\theta-\psi_i]^{-\beta}\right)^{\frac{-1}{\beta}} - \left(\frac{Q_f}{Q}\right)^{\frac{1}{\eta}} r\left(\sum_{i=1}^{n} \sigma_i[\theta-\chi_i]^{-\eta}\right)^{\frac{-1}{\eta}}$$

Thus, substituting and dividing by $r=Q^{-1/\alpha}$

$$0 = \left(\frac{Q_f}{Q}\right)^{\frac{-1}{\alpha}} - \left(\frac{Q_f}{Q}\right)^{\frac{1}{\beta}}\left(\sum_{i=1}^{n} s_i[1-\theta-\psi_i]^{-\beta}\right)^{\frac{-1}{\beta}} - \left(\frac{Q_f}{Q}\right)^{\frac{1}{\eta}}\left(\sum_{i=1}^{n} \sigma_i[\theta-\chi_i]^{-\eta}\right)^{\frac{-1}{\eta}}$$

or

$$1 = \left(\frac{Q_f}{Q}\right)^{A+\frac{1}{\beta}}\left(\sum_{i=1}^{n} s_i[1-\theta-\psi_i]^{-\beta}\right)^{\frac{-1}{\beta}} + \left(\frac{Q_f}{Q}\right)^{A+\frac{1}{\eta}}\left(\sum_{i=1}^{n} \sigma_i[\theta-\chi_i]^{-\eta}\right)^{\frac{-1}{\eta}}.$$

or

$$1=\left(\frac{Q}{Q_f}\right)^{-\left(A+\frac{1}{\beta}\right)}\left(\sum_{i=1}^{n}s_i[1-\theta-\psi_i]^{-\beta}\right)^{\frac{-1}{\beta}}+\left(\frac{Q}{Q_f}\right)^{-\left(A+\frac{1}{\eta}\right)}\left(\sum_{i=1}^{n}\sigma_i[\theta-\chi_i]^{-\eta}\right)^{\frac{-1}{\eta}}.$$

This equation can be solved for the ratio Q_f/Q which yields the underproduction.

Proof of Theorem 7: First, we need to define a couple of elasticities. Differentiating the equilibrium condition

$$Q^{\frac{1}{\alpha}} = \left(\frac{Q}{\Gamma}\right)^{\frac{1}{\eta}}$$

with respect to Γ yields the supply elasticity

$$\frac{\Gamma}{Q}\frac{\partial Q}{\partial \Gamma} = \frac{\alpha}{\eta + \alpha}.$$

Similarly, define the equilibrium price elasticity

$$\frac{\Gamma}{p}\frac{\partial p}{\partial \Gamma} = \frac{-1}{\eta + \alpha}.$$

Differentiating firm i's profits with respect to its capacity report and substituting the above elasticities into the first order condition, we obtain

$$\begin{aligned} 0 &= \frac{\partial p}{\partial \Gamma}[Q\sigma_i - q_i] + p\frac{\partial Q}{\partial \Gamma}\sigma_i + pQ\left[\frac{1}{\Gamma} - \frac{\widehat{v}_i}{\Gamma}\right] - c_i'\left[\left(\frac{1}{\Gamma} - \frac{\widehat{v}_i}{\Gamma}\right)X + \sigma_i\frac{\partial Q}{\partial \Gamma}\right] \\ &= \frac{\partial p}{\partial \Gamma}\frac{\Gamma}{p}[\sigma_i - s_i] + \frac{p - c_i'}{p}\left[\frac{\partial Q}{\partial \Gamma}\frac{\Gamma}{Q}\sigma_i + (1 - \sigma_i)\right] \\ &= \frac{-(\sigma_i - s_i)}{\eta + \alpha}. + \frac{(p - c_i')}{p}\left[\frac{\eta(1 - \sigma_i) + \alpha}{\eta + \alpha}\right]. \end{aligned}$$

Q.E.D.

Proof of Theorem 9: Let $\widehat{v}_i^*$ denote the equilibrium pre-merger reports and let $\Gamma^*_{-12} = \sum_{i=2}^{n} \widehat{v}_i^*$ denote the aggregate capacity reported by sellers 2 through n. Define $z_{12} = \frac{\widehat{v}_{12}}{v_1}$ and assume, without loss of generality, that $v_1 \geq v_2$. Fixing its rival reports to their pre-merger levels, the merged firm's equilibrium best reply solves

$$\frac{z_{12}}{(\nu_1 + v_2)^{-1}\Gamma^*_{-12} + z_{12}}\left(\frac{\varepsilon\eta + \alpha\eta}{\varepsilon\eta + \alpha\eta + \alpha\varepsilon}\right) = \frac{1 - z_{12}^{\frac{1}{\eta}}}{\left[\frac{\eta+1}{\eta} - z_{12}^{\frac{1}{\eta}}\right]}.$$

The fact that

$$(\nu_1 + v_2)^{-1}\Gamma^*_{-12} < v_1^{-1}\left(\Gamma^*_{-12} + \widehat{v}_2^*\right)$$

implies that $z_{12} < \frac{\widehat{v}_1^*}{v_1}$. Hence,

$$\frac{\widehat{v}_{12}}{\nu_1 + v_2} < \frac{\widehat{v}_1^*}{v_1} \Longrightarrow \widehat{v}_{12} < \frac{\widehat{v}_1^*}{v_1}(v_1 + v_2) < \widehat{v}_1^* + \widehat{v}_2^*$$

where the last inequality follows from Lemma 1. Thus, the merging firms best reply to its rival's pre-merger capacity reports is to reduce its reported total capacity. The result then follows from Lemma 8. **Q.E.D.**

Proof of Theorem 10:

Consider a vertical merger between a seller, firm 1, and a buyer, firm 2, in a vertically separated market. Using market clearing conditions to solve for $Q(\Gamma, K)$, the optimization problem for the vertically integrated firm can be expressed as follows:

$$\max_{\widehat{\nu}_{12},\widehat{k}_{12}} \Gamma^{\frac{-(\eta+1)}{\varepsilon+\eta}} K^{\frac{\eta+1}{\varepsilon+\eta}} \left[\widehat{\nu}_{12} - \left(\frac{\eta}{\eta+1}\right) \nu_1^{-\frac{1}{\eta}} \widehat{\nu}_{12}^{\frac{\eta+1}{\eta}}\right] + \Gamma^{\frac{\varepsilon-1}{\varepsilon+\eta}} K^{-\frac{(\varepsilon-1)}{\varepsilon+\eta}} \left[\left(\frac{\varepsilon}{\varepsilon-1}\right) k_2^{\frac{1}{\varepsilon}} \widehat{k}_{12}^{\frac{\varepsilon-1}{\varepsilon}} - \widehat{k}_{12}\right].$$

The first-order conditions can be expressed as follows:

$$\left(\frac{\eta}{\varepsilon+\eta}\right) \frac{\widehat{\nu}_{12}}{\Gamma} \left[\frac{\eta+1}{\eta} - \widehat{\nu}_{12}^{\frac{1}{\eta}} \nu_1^{-\frac{1}{\eta}}\right] - \left[1 - \left(\frac{\widehat{\nu}_{12}}{\nu_1}\right)^{\frac{1}{\eta}}\right] = \left(\frac{\varepsilon}{\varepsilon+\eta}\right) \frac{\widehat{k}_{12}}{K} \left[k_2^{\frac{1}{\varepsilon}} \widehat{k}_{12}^{-\frac{1}{\varepsilon}} - \left(\frac{\varepsilon-1}{\varepsilon}\right)\right]$$

$$\left(\frac{\varepsilon}{\varepsilon+\eta}\right) \frac{\widehat{k}_{12}}{K} \left[k_2^{\frac{1}{\varepsilon}} \widehat{k}_{12}^{-\frac{1}{\varepsilon}} - \left(\frac{\varepsilon-1}{\varepsilon}\right)\right] - \left[k_2^{\frac{1}{\varepsilon}} \widehat{k}_{12}^{-\frac{1}{\varepsilon}} - 1\right] = \left(\frac{\eta}{\varepsilon+\eta}\right) \frac{\widehat{\nu}_{12}}{\Gamma} \left[\frac{\eta+1}{\eta} - \nu_1^{-\frac{1}{\eta}} \widehat{\nu}_{12}^{\frac{1}{\eta}}\right].$$

Combining the two equations, we find that

$$1 - \left(\frac{\widehat{\nu}_{12}}{\nu_1}\right)^{\frac{1}{\eta}} = -\left[\left(\frac{k_2}{\widehat{k}_{12}}\right)^{\frac{1}{\varepsilon}} - 1\right].$$

Thus, the best reply of the vertically integrated firm is to understate its capacity on one side of the market and overstate its capacity on the other side of the market. Solving then yields

$$\frac{k_2}{\widehat{k}_{12}} = \left(\frac{\widehat{\nu}_{12}}{\nu_1}\right)^{\frac{\varepsilon}{\eta}}.$$

Substitutiting out the consumption capacity ratio, the vertically integrated first order conditions reduces to a single equation

$$\left(\frac{\eta}{\varepsilon+\eta}\right) \left(\frac{z_{12}}{v_1^{-1}\Gamma_{-12} + z_{12}}\right) \left[\frac{\eta+1}{\eta} - z_{12}^{\frac{1}{\eta}}\right] - \left[1 - z_{12}^{\frac{1}{\eta}}\right] = \frac{\left(\frac{\varepsilon}{\varepsilon+\eta}\right) z_{12}^{-\frac{\varepsilon}{\eta}}}{k_2^{-1} K_{-12} + z_{12}^{-\frac{\varepsilon}{\eta}}} \left[z_{12}^{\frac{1}{\eta}} - \left(\frac{\varepsilon-1}{\varepsilon}\right)\right]$$

Let H denote the LHS expression and G denote the RHS. It is easily verified that both functions are strictly increasing in z on the unit interval. We denote post-merger equilibrium reports using the superscript **.

(i) If there is only one seller, then

$$H(z) = -\left(\frac{\varepsilon - 1}{\varepsilon + \eta}\right) + \left(\frac{\varepsilon}{\varepsilon + \eta}\right) z^{\frac{1}{\eta}}.$$

Let z_1^* denote the equilibrium report of firm 1 prior to the merger. It solves $H(z) = 0$ which implies

$$z_1^* = \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\eta}.$$

But then $G(z_1^*, K_{-12}) = 0$, so $z_{12}^{**} = z_1^*$. Furthermore, by Lemma 7, the solution is independent of the independent buyers' capacity reports. Thus, vertical integration has no effect on the vertically integrated firm's reported production capacity and its consumption capacity report is given by

$$\frac{k}{\widehat{k}^{**}} = \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\frac{1}{\varepsilon}}$$

which implies that $\widehat{k}^{**}$ is larger than k and hence larger than $\widehat{k}^{*}$. From Lemma 7, independent buyers respond by increasing their capacity reports. As a result, reported demand shifts out, output increases, price rises, and the market share of the independent buyers fall.

(ii) If there is only one buyer, then the solution to $H(z, \Gamma_{-12}^{**}) = G(z)$ is

$$z_{12}^{**} = \left(\frac{\eta + 1}{\eta}\right)^{\eta},$$

which implies that

$$\frac{k}{\widehat{k}^{**}} = \left(\frac{\eta + 1}{\eta}\right)^{\varepsilon}.$$

But it is easily verified that this is also the solution to the monopoly buyer's first order condition prior to the merger. Thus, the merger causes the vertically integrated firm to increase its reported production capacity but reported consumption capacity is unchanged. From Lemma 7, independent sellers respond by increasing their capacity reports. As a result, reported supply shifts out, output increases, price falls, and the market share of the independent sellers fall.

(iii) If there is at least two buyers and sellers, then the solution to the first order condition of the merged firm,

$$z_{12}^{**} \in \left[\left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\eta}, \left(\frac{\eta + 1}{\eta}\right)^{\varepsilon}\right]$$

since

$$H\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\eta}, \Gamma^{**}_{-12}\right) < G\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\eta}, K^{**}_{-12}\right) = 0$$

and

$$H\left(\left(\frac{\eta+1}{\eta}\right)^{\varepsilon}, \Gamma^{**}_{-12}\right) > G\left(\left(\frac{\eta+1}{\eta}\right)^{\varepsilon}, K^{**}_{-12}\right) > 0,$$

and both functions are increasing on this interval. Thus, $G(z^{**}_{12}, K^{**}_{-12}) > 0$ which implies that

$$H(z^{**}_{12}, \Gamma^{**}_{-12}) > 0 = H(z^{*}_{1}, \Gamma^{*}_{-12}).$$

which is consistent with Lemma 7 if and only if $z^{**}_{12} > z^{*}_{1}$ and $\Gamma^{**}_{-12} > \Gamma^{*}_{-12}$. Hence, the vertical merger causes the merged firm to increase its reported production and consumption capacity. Independent buyers and sellers also increase their reported capacity. **Q.E.D.**